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ABSTRACT

Technology has revolutionized the way mathematics and science are taught and few innovations in mathematics education have had as much impact as the graphing calculator. Fourteen classroom teachers were brought together during the summer of 1997 and during a week-long workshop, the teachers collaborated to develop a resource guide for integrating graphing calculators into the mathematics curriculum for grades 7-12. This book provides those lessons that cover a wide variety of mathematical concepts and topics and are intended to facilitate the use of the graphing calculator in the classroom. Designed to be consistent with national standards and state frameworks for mathematics, the lessons emphasize hands-on, problem solving approaches and connections to science and the real world. Some lessons also incorporate other forms of technology such as calculator-based laboratories. (ASK)

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GRAPHING CALCULATORS IN MATHEMATICS GRADES 7-12

A Resource Guide for the Classroom and for Preservice/Inservice Training

Developed by
the Center of Excellence for Science and Mathematics Education at The University of Tennessee at Martin

in Collaboration with
the Eisenhower Regional Consortium for Mathematics and Science Education at

AEL Appalachia Educational Laboratory, Inc.

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in
Mathematics Grades 7–12

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and for Preservice/Inservice Training

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About the CESME

The mission of the CESME is the improvement of teaching and learning in science and mathematics in Tennessee and nationally through research, development, and training. Focus areas are:

- improvement of science and mathematics education in the rural school community
- effective use of appropriate educational technology in science and mathematics education
- reducing underrepresentation of women and minorities in science and mathematics

The CESME conducts workshops for K–12 math and science teachers that emphasize strategies of teaching that meet national standards and are aligned with state curriculum frameworks. One area of professional development is the coordination of workshops in which teachers expand and utilize their expertise to develop innovative resources that can be used by other teachers throughout the state and beyond.

For information about other CESME projects and educational resources, contact the CESME.

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About AEL

AEL's mission is to link the knowledge from research with the wisdom from practice to improve teaching and learning. AEL serves as the Regional Educational Laboratory for Kentucky, Tennessee, Virginia, and West Virginia. For these same four states, it operates both a Regional Technology in Education Consortium and the Eisenhower Regional Consortium for Mathematics and Science Education. In addition, it serves as the Region IV Comprehensive Technical Assistance Center and operates the ERIC Clearinghouse on Rural Education and Small Schools.

The Eisenhower Regional Consortium for Mathematics and Science Education collaborates with educators to improve mathematics and science education. Specifically, the Consortium identifies and shares exemplary mathematics and science instructional materials and provides technical assistance to help teachers, administrators, and college faculty use new teaching methods and assessment strategies.

Information about AEL projects, programs, and services is available by writing or calling AEL.

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Preface

Technology has revolutionized the way mathematics and science are taught, and few innovations in mathematics education have had as much impact as the graphing calculator. Perhaps the only instruments more important in developing sound mathematical foundations are our students' minds. Armed with the ability to effectively use technology, students have the potential to accelerate their mastery of mathematical principles and techniques, and to affect their world in meaningful ways. Our challenge as teachers is to harness and exploit the graphing calculator as a tool to enhance learning.

Many of us have struggled with the role that graphing calculators should play in our classrooms. Assimilating this new educational tool into our teaching styles has not been easy for many of us who have been accustomed to teaching in more traditional ways. Yet, time and again, we see what can result from placing hand-held calculators in the hands of our students. Our reluctance to embrace the graphing calculator technology is not that we need to be convinced of its usefulness, but that we need to know more ways to use it effectively. Learning how to use the graphing calculator, in conjunction with other forms of technology and in substantial ways, is as important for teachers as it is for students. Our goal as educators is for students to learn to use technology to develop thinking skills, not merely to-punch buttons to obtain meaningless numbers and pictures. We want graphing calculators to enhance students' abilities to solve problems in multiple ways, to visualize mathematical concepts, and to find excitement in the learning process.

It was with these thoughts in mind that fourteen classroom teachers were brought together during the summer of 1997 as part of a collaborative project of the CESME and the Eisenhower Regional Consortium for Mathematics and Science Education at AEL. During a weeklong workshop, the teachers collaborated to develop a resource guide for integrating graphing calculators into the mathematics curriculum, grades 7-12. The resulting lessons cover a wide variety of mathematical concepts and topics and are intended to facilitate the use of the graphing calculator in the classroom. The lessons were planned for teachers with no experience with graphing calculators as well as for teachers who have been using graphing calculators as long as they have been available. Designed to be consistent with national standards and state frameworks for mathematics, the lessons emphasize hands-on, problem-solving approaches, and connections to science and the real world. Some incorporate other forms of technology such as the Calculator Based Lab (CBL).

Like you, we are teachers dedicated to instilling a desire to learn within our students and to structuring successful learning environments. Our hope is that you will be able to use this guide in productive ways in the classroom, in your personal professional development, and/or in conducting inservice or preservice training for other mathematics educators.

—Ron Stewart for the Authors

Important Information

Notation

- Keystrokes are indicated by bold lettering enclosed in < >. For example, **<ZOOM>**.
- An equivalent menu is enclosed in parentheses following the keystrokes. For example, **<2nd> <STAT>** (LIST) indicates pressing the key labeled 2nd and the key labeled STAT for the LIST menu.
- Appropriate student classes are indicated on the tops of the sheets by the following symbols:

Audience	Symbol
Arithmetic Middle Grades	●
Math Tech I Math Tech II	▲
Pre-Algebra Algebra I Algebra II	■
Geometry	⬟
Probability and Statistics	⬢
Advanced Algebra Pre-Calculus Calculus	★

Note: **Although TI-82 and TI-83 keystrokes are given, the lessons could easily be modified for any graphing calculator. Many hours have been spent developing and editing and checking keystrokes; however, the graphing calculator manual can always be consulted for additional help or information.**

Lesson 1: Fun With Function Families

Authors
- ★ Edith Lancaster, Central HS, Bruceton, TN
- ★ Jo Ann Simmons, Peabody HS, Trenton, TN

Approximate Time Needed for Lesson

Two to three 90-minute periods; four traditional class periods

Content Area

Patterns, Functions, and Algebraic Thinking

Process Standards

Problem Solving, Reasoning, Communication

Goals for Students

- To identify fundamental "head of family" equations and graphs
- To draw the graph given a "head of family" equation
- To write the equation given a "head of family" graph
- To use transformations to draw the graph of an equation
- To identify the transformations of a graph and write the corresponding equation

Description or Summary of Lesson

Students in Math Tech I, Algebra I, and/or Algebra II are the intended audience. Students will work individually or in small groups. The students will develop tables of functional values for four fundamental equations and plot points on coordinate grids. Students will use the TI-81, TI-82, or TI-83 for the tables and for graphing. Students will recognize and use translation, reflection, and size transformations of the graphs for four fundamental equations.

Background Information

This lesson is introductory material in preparation for transformations and practice material for transformations on "head of family" functions. Prior to this lesson students should be able to do coordinate graphing and should have basic keystroke skills on a TI-81, TI-82, or TI-83.

Materials and Equipment

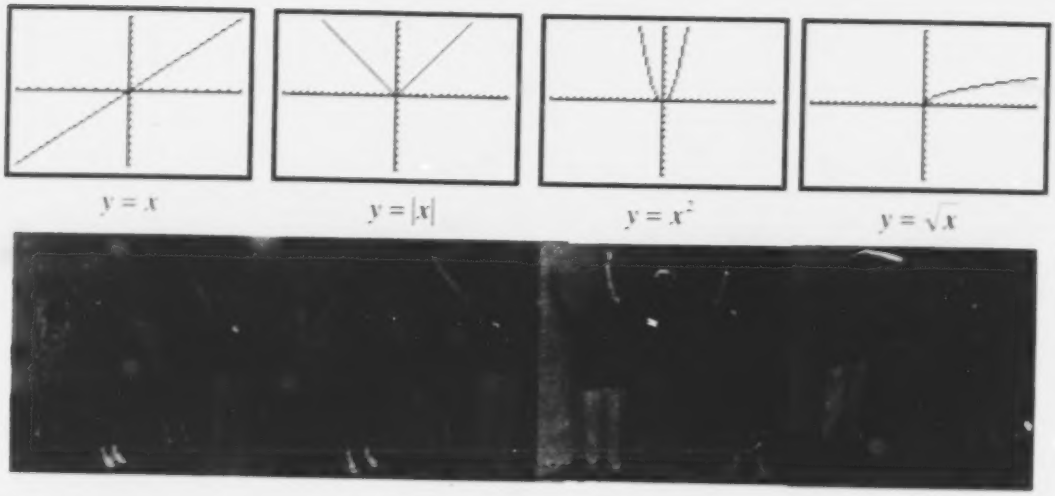
- View screen with graphing calculator
- Overhead projector, overhead graph grid, transparency sheet, overhead pens
- Each student will need:
  - 2 sheets of graph paper, one cut into four 4" x 5" pieces
  - 1 sheet of transparency paper about 4" x 5"
  - access to 1 overhead pen
  - damp paper towel
  - access to a TI-81, TI-82, or TI-83 graphing calculator
  - handouts

Activity One: Four "Head of Family" Functions

Teacher Notes

Outline:

- Review table building and coordinate graphing.
- Build tables for $y = x$, $y = |x|$, $y = x^2$, and $y = \sqrt{x}$.
- On graph paper, graph the points from the tables.
- Enter the function $y = x$ into the **<Y=>** menu on graphing calculator.
- Set window on calculator using the **<Zoom> <6>** command.
- Graph the function. Repeat the above steps for the other three functions.
- Practice shapes of fundamental graphs using "mathercise" (see below).



Note: Teachers will want to form the graphs so they look correct to the students. The pictures above are correct to the ones forming the graphs.

- The domain and range of each function should be discussed so that the students can choose appropriate values of "x" to place in a t-table. Suggested domain values to choose for $y = x$ would be integers between -10 and 10; for $y = |x|$ would be integers between -10 and 10; for $y = x^2$ would be integers between -4 and 4; and for $y = \sqrt{x}$ would be {0, 1, 4, 9, 16}.

- **<ZOOM> <5>** (ZSquare) sets the window of the calculator in the ratio of about 3:2 and since the screen is in that ratio the vertical scale and horizontal scale are the same size. This will allow the calculator graph to look like the student's graph and not appear distorted.

Activity One Handout

1. Fill in the following t-tables using appropriate values of x and graph each function on graph paper.

$y = x$		$y = \|x\|$		$y = x^2$		$y = \sqrt{x}$	
x	y	x	y	x	y	x	y

2. Using the graphing calculator (in function mode), press the **<Y=>** key. Enter $Y_1 = x$. Press **<ZOOM>**, press **<6>** to set the window to a standard grid. Press **<GRAPH>** to see the graph. Compare the shape of this graph to your graph of $y = x$.

   TI-82 and TI-83 users: Make a table of values for the function placed in $Y_1 =$ by pressing **<2nd>** **<WINDOW>** (TBLSET). Set TBLSTART = at -10 and the ΔTbl = to 1. Independent and Dependent should be on Auto. Press **<2nd>** **<GRAPH>** (TABLE) and compare the t-table you made with the table in the calculator.

   TI-81 users: Set the **<RANGE>** at Xmin = -9.6, Xmax = 9.4, xscale = 1, Ymin = -6.6, Ymax = 6.4, Yscale = 1. Press the **<TRACE>** key and compare the (x,y) values shown on the screen with those in your table.

3. Press the **<Y=>** key and enter $Y_1 = |x|$ using the following key strokes:

   TI-81/82: $Y_1 =$ **<2nd>** $< x^{-1} >$ (ABS) **<X,T,θ>**

   TI-83: $Y_1 =$ **<MATH>** $< \blacktriangleright >$ (NUM) **<ENTER>** [1:abs( ] **<X,T,θ,n>**

   Press **<ZOOM>** **<5>** (Zsquare). Press **<GRAPH>**.

   Compare the shape of this graph to your graph of $y = |x|$.

   Press **<2nd>** **<GRAPH>** (TABLE) and compare the t-table you made with the table in the calculator.

   Note: Since you changed the function in $Y_1 =$ the values in the table have changed to match the new function.

4. Repeat Step 3 using $y = x^2$ and $y = \sqrt{x}$.

5. Mathercise: Practice the shape of the fundamental equations using arm movements. Stand and demonstrate the shape of the graph when your teacher calls out an equation. Remember the shapes of the four graphs because your teacher will be calling them randomly and rapidly.

Activity Two: Transformations of the "Head of Family" Functions Using the Graphing Calculator

Teacher Notes

Outline:

- Clear the <**Y=**> menu. Use a table (Activity Two, Handout 1) to record the results of your observations. They will include, for each pair of equations, the graphs and any conclusions that can be made. Graph the "head of family" function in $Y_1 =$, and the transformed graph in $Y_2 =$. Start with $Y_1 = x$ and $Y_2 = -x$. Record the results. Clear the <**Y=**> menu. Next use $Y_1 = |x|$ and $Y_2 = -|x|$. Third use $Y_1 = x^2$ and $Y_2 = -x^2$ and last use $Y_1 = \sqrt{x}$, and $Y_2 = -\sqrt{x}$. Fill in the charts each time new equations are used and answer questions at bottom of chart.

- Clear the <**Y=**> menu. Use a table (Activity Two, Handout 2) to record your observations. This time, place the "head of family" graph in $Y_1 =$ and *twice* the function in $Y_2 =$ ($2x$, $2|x|$, $2x^2$, and $2\sqrt{x}$ respectively).

- Repeat using $Y_2 =$ ($\frac{x}{2}$, $\frac{|x|}{2}$, $\frac{x^2}{2}$, $\frac{\sqrt{x}}{2}$) and recording in Activity Two, Handout 3.

- Repeat using $Y_2 =$ ($x+2$, $|x|+2$, x^2+2, $\sqrt{x}+2$) and recording in Activity Two, Handout 4.

- Repeat using $Y_2 =$ ($x-2$, $|x|-2$, x^2-2, $\sqrt{x}-2$) and recording in Activity Two, Handout 5.

- Complete Activity Two, Handout 6, and draw conclusions in paragraph form.

Note: For Activity Two, Handout 1, you may want to emphasize that the line of reflection is the x-axis.

Activity Two, Handout 1

- Using your graphing calculator (in function mode, sequential mode) press **<Y=>**. $Y_1 = x$. Enter $Y_2 = -x$. Press **<ZOOM>**, press **<4>**. Press **<GRAPH>**. Enter resulting information in table below as #1.

  Go back to **<Y=>**. Clear $Y_1 =$ and enter $Y_1 = |x|$ as $Y_1 =$ abs (x). Clear $Y_2 =$ and enter $Y_2 = -|x|$ as $Y_2 =$ - abs(x). Press **<GRAPH>**. Enter resulting information in table below as #2.

  Go back to **<Y=>** . Clear Y_1 and enter $Y_1 = x^2$. Clear $Y_2 =$ and enter $Y_2 = -x^2$. Press **<GRAPH>**. Enter resulting information in table below as #3.

  Go back to **<Y=>**. Clear $Y_1 =$ and enter $Y_1 = \sqrt{x}$. Clear $Y_2 =$ and enter $Y_2 = -\sqrt{x}$. Press **<GRAPH>**. Enter resulting information in table below as #4.

	Y_1 Equation	Y_2 Equation	Sketch and Label Both Graphs	How is Y_2 Different from Y_1 ?
1.				
2.				
3.				
4.				

- What effect does replacing the function with the opposite of the function have on the graph?
- Would this transformation be called a translation, a reflection, or a vertical compression/stretch?

Activity Two, Handout 2

- Repeat instructions given in Activity Two, Handout 1, using $Y_2 = 2x$, $2|x|$, $2x^2$, and $2\sqrt{x}$.

	Y_1 Equation	Y_2 Equation	Sketch and Label Both Graphs	How is Y_2 Different from Y_1?
1.				
2.				
3.				
4.				

- What effect does replacing the function with twice the function have on the graph?

Activity Two, Handout 3

- Repeat instructions given in Activity Two, Handout 1, using $Y_2 = \frac{x}{2}, \frac{|x|}{2}, \frac{x^2}{2}$, and $\frac{\sqrt{x}}{2}$.

	Y_1 Equation	Y_2 Equation	Sketch and Label Both Graphs	How is Y_2 Different from Y_1?
1.				
2.				
3.				
4.				

- What effect does replacing the function with "the function divided by 2" have on the graph?

- Would you call the transformations in #2 and #3 translations, reflections, or vertical compressions/stretches?

Activity Two, Handout 4

- Repeat instructions given in Activity Two; Handout 1, using $= x + 2$, $y = |x| + 2$ Y2, $y = x^2 + 2$, and $y = \sqrt{x} + 2$.

Y_1 Equation	Y_2 Equation	Sketch and Label Both Graphs	How is Y_2 Different from Y_1?
1.			
2.			
3.			
4.			

- What effect does replacing the function with "the function + 2" have on the graph?

Activity Two, Handout 5

- Repeat instructions given in Activity Two, Handout 1, using $Y_2 = x - 2$, $|x| - 2$, $x^2 - 2$, and $\sqrt{x} - 2$.

	Y_1 Equation	Y_2 Equation	Sketch and Label Both Graphs	How is Y_2 Different from Y_1?
1.				
2.				
3.				
4.				

- What effect does replacing the function with "the function - 2" have on the graph?

- Would you call the transformations in #4 and #5 translations, reflections, or vertical compressions/stretches?

Activity Two, Handout 6

Explorations and Conclusions:

Choose the "head of family" graph for each of the following equations. How does the graph of each of the 10 equations below differ from its "head of family" graph? Make a conjecture for each equation and check with your calculator.

	Conjecture	Actual
1. $y = 5x$		
2. $y = 5x + 6$		
3. $y = -x + 9$		
4. $y = \|x\| - 3$		
5. $y = \sqrt{x} + 4$		
6. $y = 2x^2 - 1$		
7. $y = -x^2 + 3$		
8. $y = -2\sqrt{x}$		
9. $y = 3\|x\| + 2$		
10. $y = -3x^2$		

- What conclusions can you make? Write your conclusions in paragraph form.

Activity Three: "Hands on" Transformations

Prerequisites:
Students must have completed Activity 1 and Activity 2.

Instructions for Teacher:
Instruct students to take one piece of the graph paper and draw an x-axis and y-axis with the origin at the center of the paper. Illustrate, using the overhead projector. Have them draw one of the "head of family" graphs on this coordinate axis with a pen or pencil—the graph of $y = |x|$ is suggested. Let them place the transparency sheet over this graph and trace the "V" on the transparency sheet with an overhead pen. Have the students move the transparency sheet to show the location of the following graphs:

1. $y = -|x|$
2. $y = |x| + 3$
3. $y = |x| - 1$
4. $y = -|x| - 2$
5. $y = -|x| + 4$

- Have student-volunteer locate the new graph on the overhead while teacher walks around the room to see if the students are making the proper "flips" and "slides."
- Slide and/or flip the transparency on the overhead to some location on the grid and have students write the equation. Let one student write an equation and place the transparency at the proper location on the overhead. Have the other students write the equation. Check to see that the equations are alike. Repeat with other students selecting the equation and location.
- Let the students erase their transparency with the damp paper towel. Place the transparency over the "head of family" graph and, using the overhead pen, graph $y = 2|x|$ on the transparency. Have the students move the transparency to show the location of each of the following graphs:

  1. $y = -2|x|$
  2. $y = 2|x| + 3$
  3. $y = 2|x| - 1$
  4. $y = -2|x| + 2$

- Repeat the exercise of placing the transparency at a particular location and letting the students write the equations.

- Select another "head of the family" graph. Use a second piece of graph paper to draw its graph. Repeat the exercises, first giving the equation and locating the graph, then locating the graph and writing the equation.

Adaptations:

- These exercises could be used in Algebra II and upper level courses by adding other functions such as $y = x^3$; $y = 2^x$ or other exponential functions; $y = [x]$, the step function; $y = \log x$ or other logarithmic functions; and $y = \sin x$ or other trigonometric functions.

- Horizontal translations of the form $y = f(x + h)$ and $y = f(x - h)$ can be added to the vertical translations of the form $y = f(x) + k$ and $y = f(x) - k$.

- Reflections in the y-axis of the form $y = f(-x)$ could be added.

- A different approach would be to have a "station activity" for the graph-to-equation activity. Set up 10-12 stations around the classroom and have a graph at each station that will illustrate the transformations on the parent graphs. Let each student have 15-30 seconds at each station to write the correct equation.

- Within the resources listed are real-world problems related to function transformations.

- If a Calculator Based Lab and motion detector are available, students might enjoy walking in front of the motion detector to create each "head of family" graph.

Suggested Teacher Resources:

CBL-Explorations in Algebra for TI-82 and TI-83, by Stephen D. Nichols, Meridian Creative Group, 1996.

Graphing Calculator Activities, by Edwin Andersen and Charles Lund, Addison-Wesley Publishing, 1992.

Transforming Functions to Fit Data, by Ronald J. Carlson and Mary Jean Winter, Key Curriculum Press, 1998.

TI-83 or TI-82 Mini-Labs: Algebraic Investigations, by Phil DeMarois, Mathware, 1996.

Lesson 2: Putting Inequalities to the Test

Authors:
★ Vanessa Moore
Germantown HS, Germantown, TN
★ Sherry Morton
Germantown HS, Germantown, TN

Approximate Time Needed for Lesson:
One traditional class period

Content Area:
Patterns, Functions, and Algebraic Thinking

Process Standards:
Problem Solving, Connections

Goals for Students

- To solve linear inequalities using both calculator and non-calculator methods.
- To apply inequalities in solving real-world problems.

Description or Summary of Lesson

Students in Algebra I, Algebra II, and Pre-calculus are the intended audience. The first activity uses a sample problem to illustrate a calculator method for graphic solutions to linear inequalities. The second activity includes applying this method of solving inequalities in a real-world problem situation and relates algebraic and graphic solutions of linear inequalities.

Background Information

This lesson is introductory material with several suggestions for using advanced topics, enrichment, and discussion. Prior to this lesson students should have studied graphing lines, solving linear inequalities, and basic keystrokes on a TI-81, TI-82, or TI-83.

Materials and Equipment

- View screen with graphing calculator
- Overhead projector, overhead graph grid, transparency sheet, overhead pens
- Each student will need:
  - access to a TI-82 or TI-83 graphing calculator
  - graph paper
  - handouts

Activity One: The Graphing Calculator as a Tool for Solving One-Variable Inequalities

Teacher Notes

Outline:

- Review graphing linear inequalities.
- Review solving inequalities in one variable.
- Use the graphing calculator to see a graphic interpretation of the solution set of a one-variable inequality.

TI-82 Keystrokes

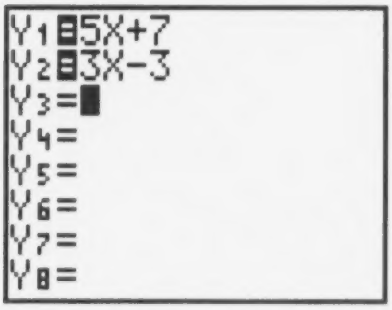
Note: The x stored value must not be zero.

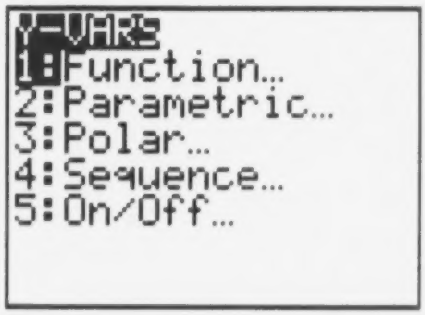
- The inequality expression must be entered in three parts on the <**Y=**> screen.
- Press <**Y=**>.

  Y_1 = type the left side of the expression.

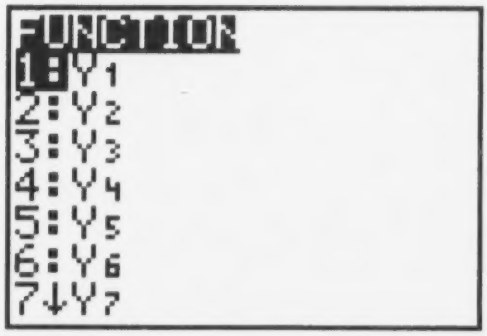
  Y_2 = type the right side of the expression.

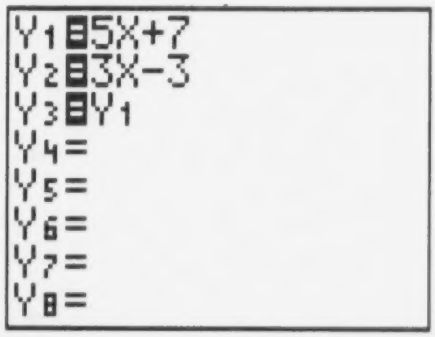
  Y_3 = put the cursor on (this is where the inequality will be written in terms of Y_1 and Y_2).

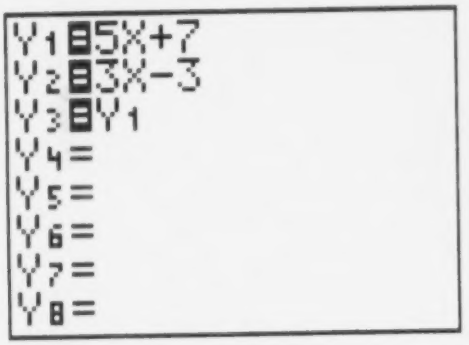




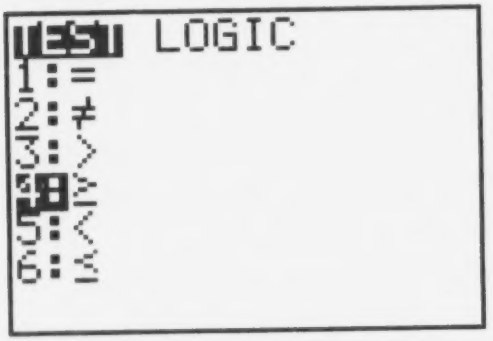
- Press <**2nd**> <**VARS**>. Highlight 1: Function. Press <**ENTER**>. Highlight 1: Y_1 Press <**ENTER**>.

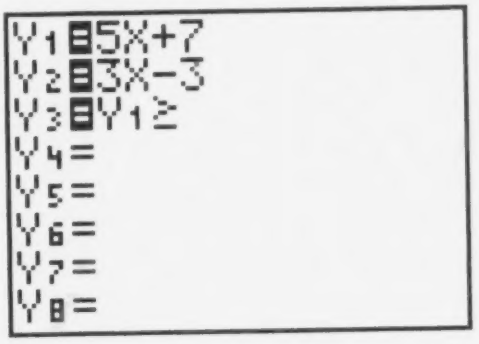




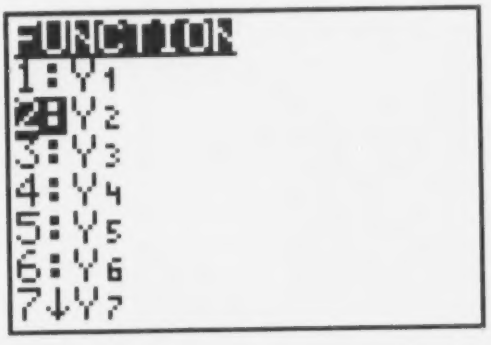


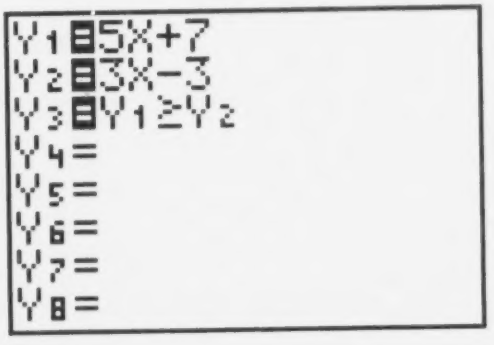
- Press <**2nd**> <**MATH**> (Test) Cursor down to the desired symbol. Then press <**ENTER**>.





- Press <**2nd**> <**VARS**>. Highlight 1: Function Press <**ENTER**>. Highlight 2: Y_2 and press <**ENTER**>.

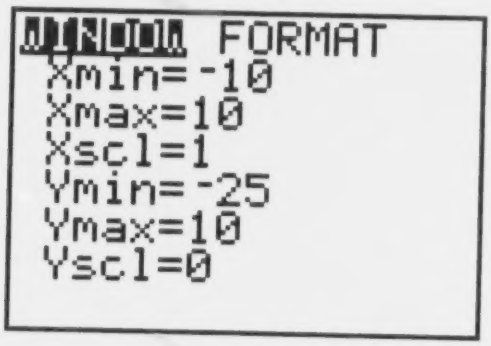




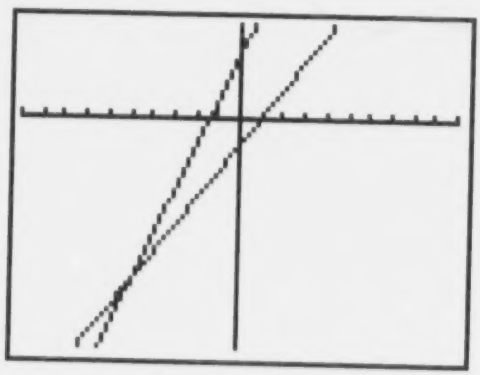
- Now is a good time to discuss domain and range in order to set the axis on the graph screen to achieve an adequate picture of the expressions in the Y_1 and Y_2. Look at the Y_1 and Y_2 and discuss what would be appropriate x and y maximums and minimums.

- Graphing calculator users can graph one, some, or all of the expressions on the $Y=$ screen. On the $Y=$ screen, if the = sign is highlighted, then the calculator will graph the expression. The expression is said to be "turned on." All three expressions in the above right screen are "turned on." To turn an expression off, move the cursor, by using the blue arrows, to the = sign of the expression desired, and press <**ENTER**>.

- Turn the Y_3 expression off in order to view the graph of the other two expressions.
- Set the viewing window in order to achieve the desired graph.
- Press <**WINDOW**>.
- Xmin (whatever desired) <**ENTER**>.
- Xmax (whatever desired) <**ENTER**>.
- Xscl (for a large screen may use a scl of 0) <**ENTER**>.
- Ymin (whatever desired) <**ENTER**>.
- Ymax (whatever desired) <**ENTER**>.
- Yscl (for a large screen, one may use a scl of 0, or the same scl as X, or different) <**ENTER**>.



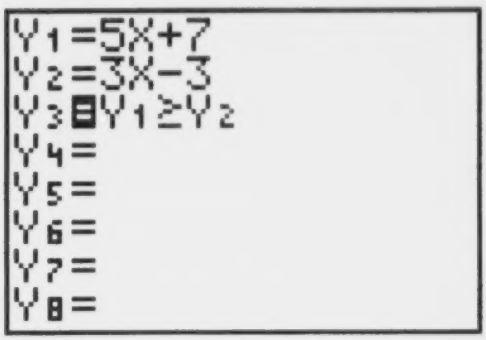
- Now press <**GRAPH**> to see the graph of Y_1 and Y_2.

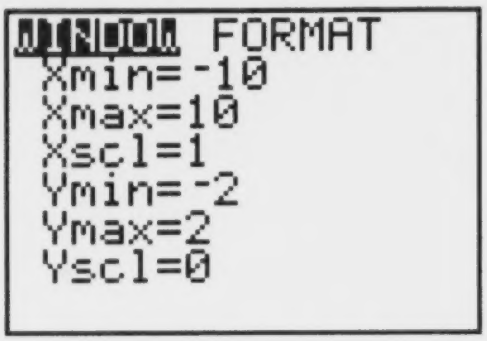


Discussion:

At this point the instructor may want to spend some time discussing the graph of the two expressions and what may occur with the inequality.

- Turn off the expressions Y_1 and Y_2 (use the method previously described).
- Turn on the expression Y_3.
- Press <**GRAPH**>. Depending on the inequality the user may need to reset the window.

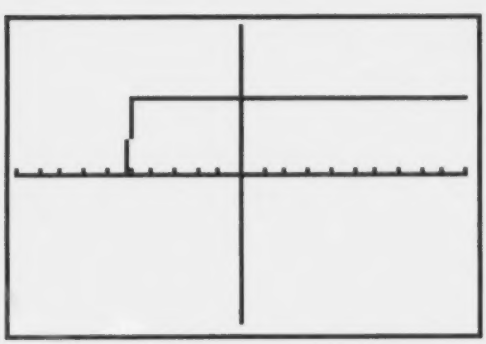


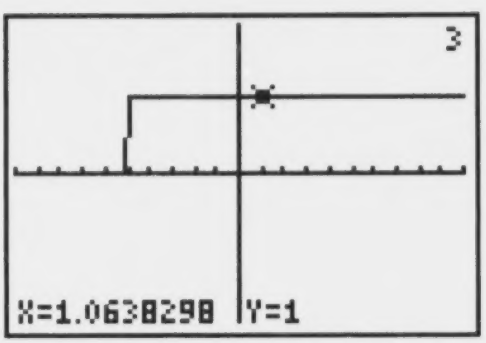


To Run the Actual Test Feature:

- Press <**TRACE**>. Move the cursor left and right. Notice the *Y*= value is either 0 or 1. If *Y* = 0 then the *x*-value does not satisfy the inequality; if *Y*= 1 then the *x*-value does satisfy the inequality.

- **Since the *Y*= value must be 0 or 1, the *Y* maximum and minimum range must include 0 and 1. This will be an important factor to remember not only in the everyday use of this feature but also in the application problem to be explored.**





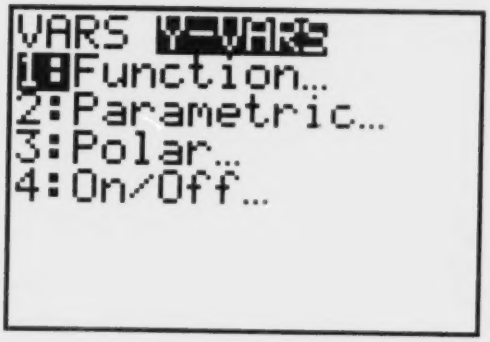
- The screen on the left is the graph of the inequality before the trace key is pressed. Notice in the upper right corner of the right screen the number 3. This number tells the user the calculator is graphing the Y_3 expression. Since the *Y* = value in the right screen is 1, the *x*-value makes the inequality true and is, therefore, in the solution set.

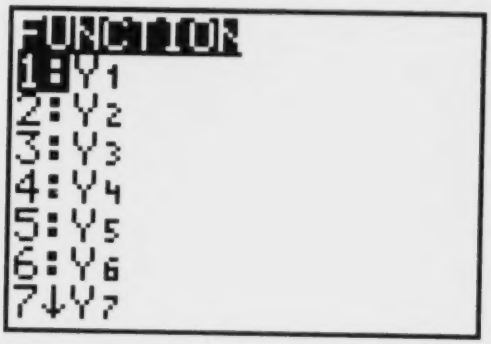
- Take time with the class to trace along the graph and make some generalizations about the values of *x* and *y* for the inequality.

TI-83 Keystrokes

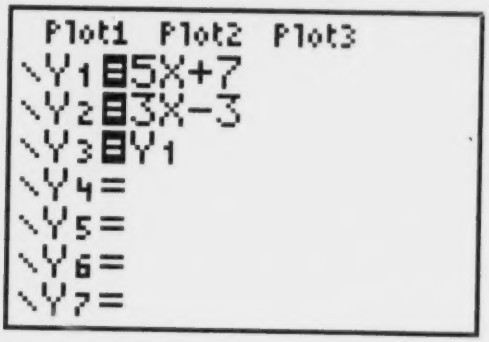
- The keystrokes are the same as the TI-82 except in the <**Y-VARS**> section. Repeat the steps from the TI-82 in order to enter the expressions into Y_1 and Y_2.

- To write Y_3 in terms of Y_1 and Y_2, press <**Y=**> and put the cursor on Y_3. Press <**VARS**> and move the cursor to highlight **Y-VARS** (left screen). Press <**ENTER**>.

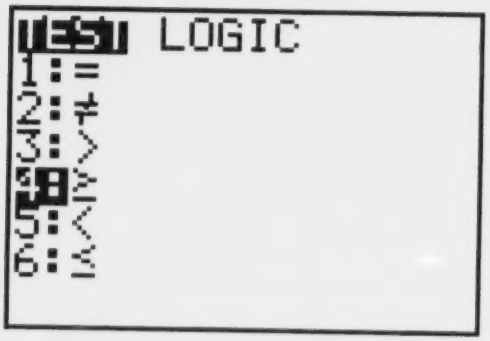


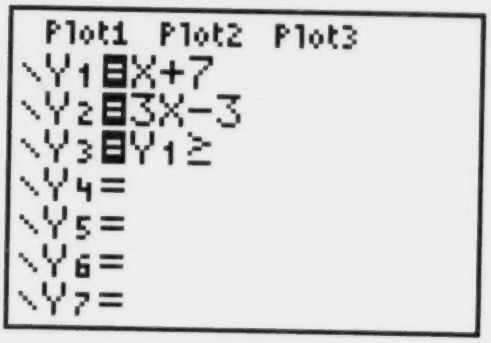


- Notice the 1 is highlighted for Y_1. Press <ENTER>.

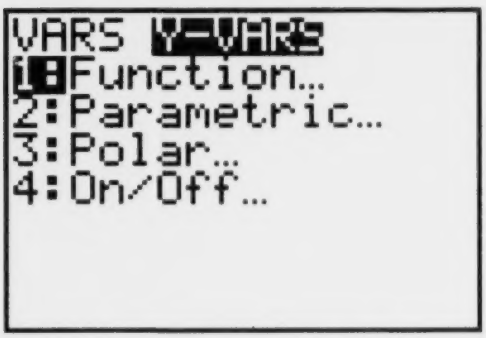


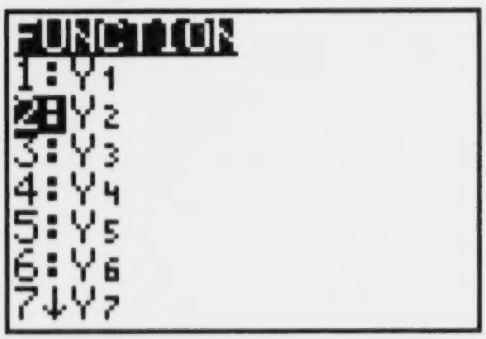
- Press <2nd> <MATH> (TEST). Highlight the desired sign (left screen), press <ENTER> (right screen).





- Press <**VARS**>, highlight Y-VARS. Press <**ENTER**>.
- 1: Function is highlighted (as before) shown on the left screen. Press <**ENTER**>.
- Highlight the Y_2 as shown on the right screen.





- Press <**ENTER**> to complete the inequality in the Y_3 expression.

To finish the testing on the TI-83 follow the same method as with the TI-82. The TI-83 has the same capability of turning an expression on or off to graph it. The three expressions in the screen above are all on.

The TI-83 has a distinct property that the TI-82 does not allow. Once in the trace mode, the user can put any desired value in for x and it will calculate the y. Take a few moments after running the testing feature and try substituting values for x, then evaluate the y. This can be done repeatedly. This also allows the user to experiment with the method of Guess and Test.

Activity Two: Application of Inequalities

Teacher Notes

Outline:

- Give students Handout 1 to complete the application problem below.
- Give students Handout 2 to complete the application problem below.

Application Problem:

- You have been nominated to plan your senior class trip. **YEA!!!!!**
- The class decided to take a cruise to the Bahamas OH YEAH!!, and only two cruise lines offer group packages.
- The Party Line has a flat fee of $25,000 and an additional $175.00 per person, with a minimum of 250 people and a maximum of 500 people.

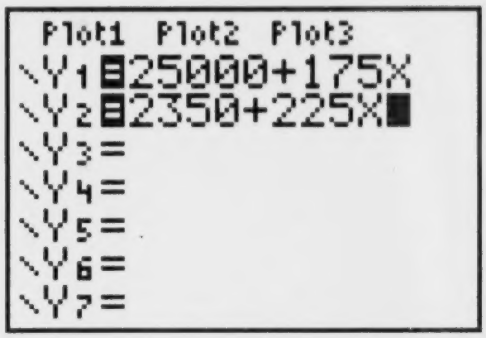
  $(68{,}750 \leq 25{,}000 + 175.00X \leq 112{,}500)$
- The Sun Ship has a flat fee of $2350 and an additional $225.00 per person, with a minimum of 300 people and a maximum of 750.

  $(69{,}850 \leq 2350.00 + 225.00X \leq 171{,}100)$

Questions:

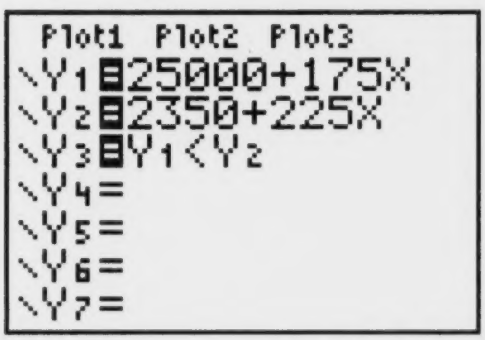
1. How many people would make the Party Line package the better offer?
2. How many people would make the Sun Ship package the better offer?
3. How many people would make the two packages equal?

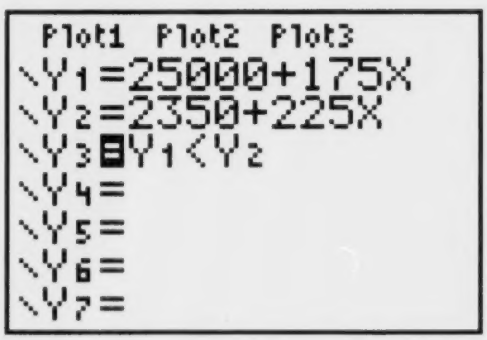
- The following screens are from the TI-83, which can be used to explore the problem.
- Enter the two expressions into the $Y =$ screen.



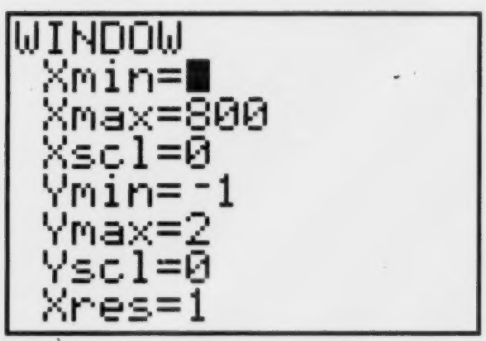
For Question 1:

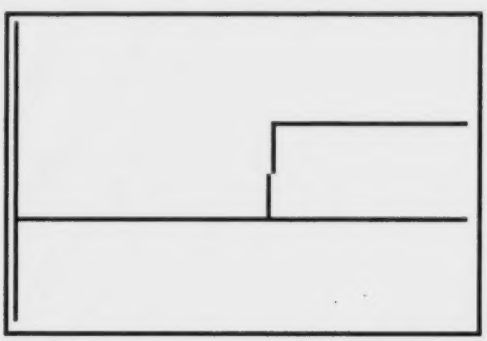
- Enter Y_3 as $Y_1 <$ (left screen). Turn off Y_1 and Y_2 (right screen)



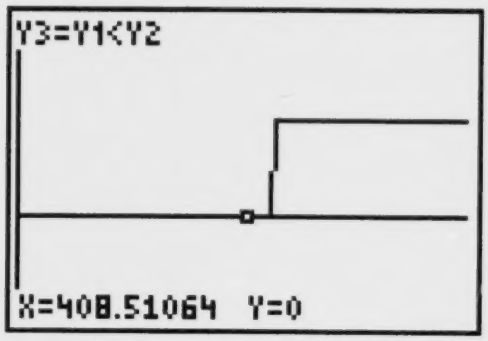


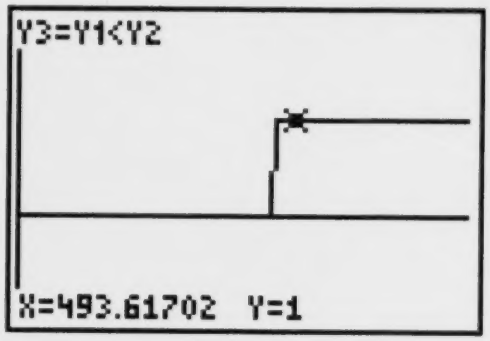
- Before graphing the inequality, **CHECK THE VIEW WINDOW**. Remember the $Y =$ value on the Test feature must be inclusive of 0 and 1. A sample window is on the left below.
- Press <**GRAPH**> (the screen on the right shows the graph before <**TRACE**> is pressed).





- Now run the test as described previously by pressing <**TRACE**> and explore the values.



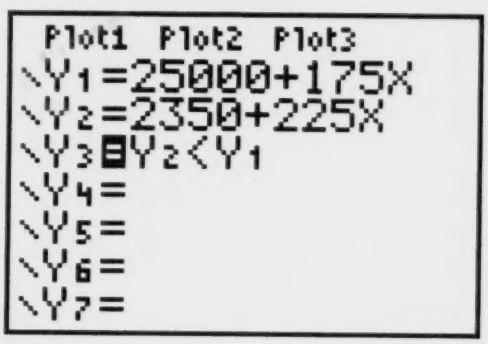


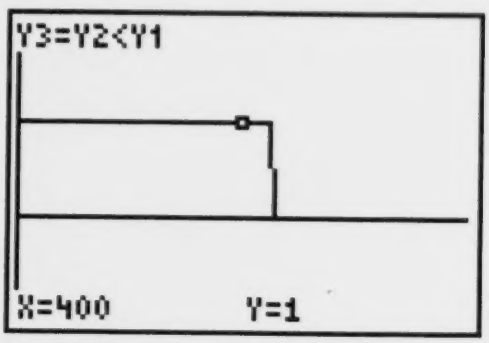
- Notice on the TI-83 the equation being graphed is in the upper left corner. This is an option that can be removed by pressing <**2nd**> <**ZOOM**> (FORMAT), and then move the cursor down, <▽>, to LabelOn or LabelOff, whichever is desired, and press <**ENTER**>.

- As stated before, a specific value may be substituted for *x* on the **TRACE** feature and the inequality evaluated. Take some time to examine this feature.

For Question 2:

- The only item that needs to be changed is the Y_3, which should now be $Y_2 < Y_1$.
- Follow the same procedure as for Question 1.
- Check the window.
- Graph and Trace.

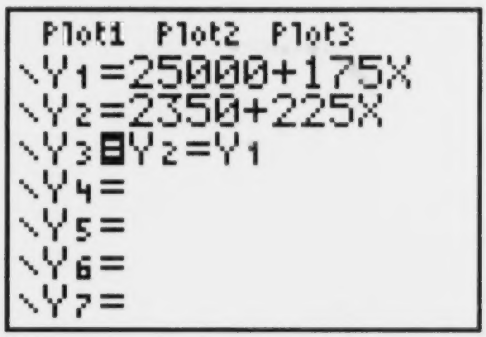




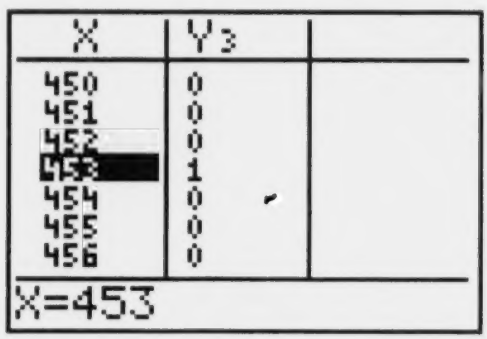
- Take time to discuss the results found. Compare the two questions explored.

Question 3:

- Replace Y_3 with $Y_1 = Y_2$. This can be done by following the procedure previously discussed.
- Since the equality would be only one point, the graphing method of testing will not be a working method. At this time it would be better to use the Table feature. As done previously, turn off Y_1 and Y_2 and turn on Y_3.



- Since the inequalities have been discussed the students have an idea of the values that will satisfy the equation.
- To use the Table feature: Press <**2nd**> <**WINDOW**> (TBLSET) Press <**ENTER**>.
- Set TBLStart =450 and Δ Tbl= 1.



- Press <**2nd**> <**GRAPH**> (TABLE).

- Notice all of the values have a *Y* value of 0 except 453. The *Y* value for 453 is 1, therefore 453 is the number of people that will make the equation true.

- At this time the user should be able to apply the information of this activity to solve any inequality.

- Have students complete the Activity Two Handout.

Activity Two Handout

Application Problem:

- You have been nominated to plan your Senior Class Trip. **YEA!!!!!**
- The class decided to take a cruise to the Bahamas [illegible], and only two cruise lines offer group packages.
- The Party Line has a flat fee of $25,000 and an additional $175.00 per person, with a minimum of 250 people and a maximum of 500 people.

1. Write an inequality to represent this fee structure.

- The Sun Ship has a flat fee of $2350 and an additional $225.00 per person, with a minimum of 300 people and a maximum of 750.

2. Write an inequality to represent this fee structure.

3. Use your graphing calculator to answer the following questions:

   a. How many people would make the Party Line package the better offer?

   b. How many people would make the Sun Ship package the better offer?

   c. How many people would make the two packages equal?

4. Find the solutions of the following inequalities graphically and algebraically.

a. $3(4x + 7) < 21$

Sketch of Graphic Solution	Algebraic Solution

b. $2(3x + 4) - 2 \leq 3(1 + 3x)$

Sketch of Graphic Solution	Algebraic Solution

c. $3x - 2(x - 5) > 2(x + 4)$

Sketch of Graphic Solution	Algebraic Solution

Lesson 3: Solving Inequalities on the TI-83

Authors

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Approximate Time Needed For Lesson
One to two traditional class periods or one 90-minute period

Content Area
Patterns, Functions, and Algebraic Thinking

Process Standards
Reasoning and Connections within Mathematics

Goals for Students

- To solve inequalities graphically
- To algebraically interpret graphic solutions of inequalities

Description or Summary of Lesson

Students in Algebra I or Algebra II are the intended audience. The lesson consists of four activities that, depending upon the students' calculator mastery, may be teacher directed or structured for cooperative groups. The first activity focuses on single-variable inequalities, the second on absolute value inequalities, and the final two extend to two-variable and compound inequalities. The activities include examples as well as practice exercises.

Background Information

This lesson is an introduction to solving inequalities on the TI-83 graphing calculator. Prior to the lesson, students should already have studied the algebraic solution of simple inequalities.

Materials and Equipment

- TI-83 graphing calculator with view screen
- Overhead projector
- Each student will need:
  - access to a TI-83 graphing calculator
  - handouts

Activities One to Four

Teacher Notes

Outline:

- Teacher leads students through the example in Activity One: Single-Variable Inequalities Handout.
- Working in cooperative groups, students complete Activity One Handout.
- Teacher leads students through the example in Activity Two: Absolute Value Inequalities Handout.
- Working in cooperative groups, students complete Activity Two Handout.
- Teacher leads students through the example in Activity Three: Graphing Inequalities in Two Variables Handout.
- Working in cooperative groups, students complete Activity Three Handout.
- Teacher leads students through the example in Activity Four: Compound Inequalities Handout.
- Working in cooperative groups, students complete Activity Four Handout.

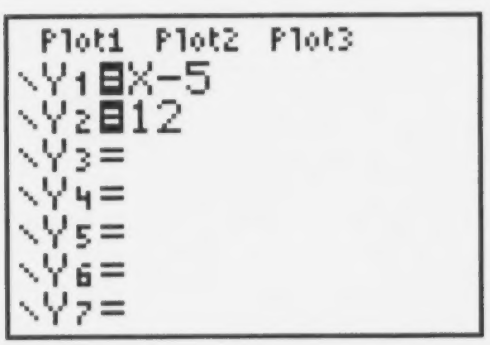
Activity One Handout: Single-Variable Inequalities

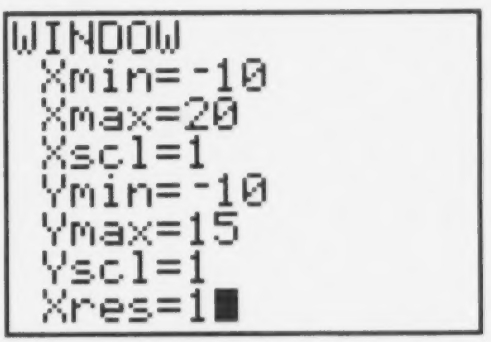
Example:

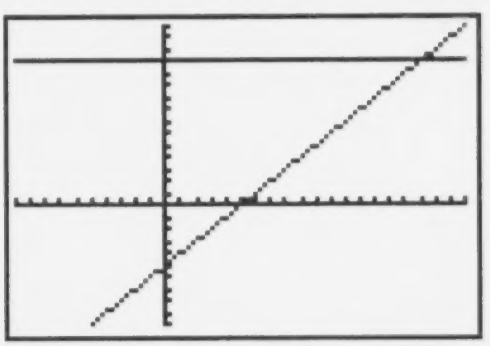
$x - 5 < 12$

- In function mode (<**MODE**> and highlight FUNC), press <**Y=**> and enter the left side of the inequality in Y_1, the right side in Y_2. Set an appropriate <**WINDOW**> so that any point(s) of intersection show and press <**GRAPH**>.

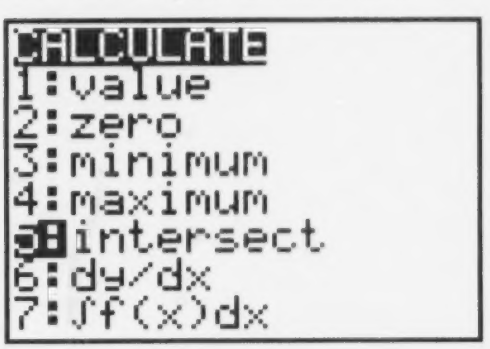
  Note: Be sure plots are turned off by <**2nd**> <**STAT PLOT**> <**4**>.

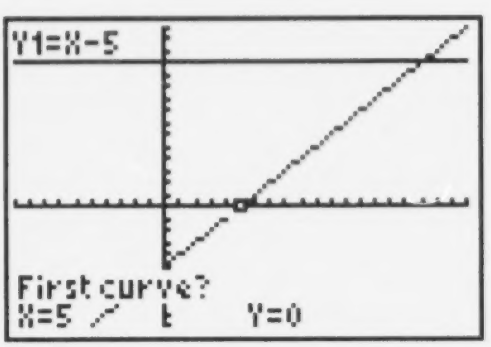


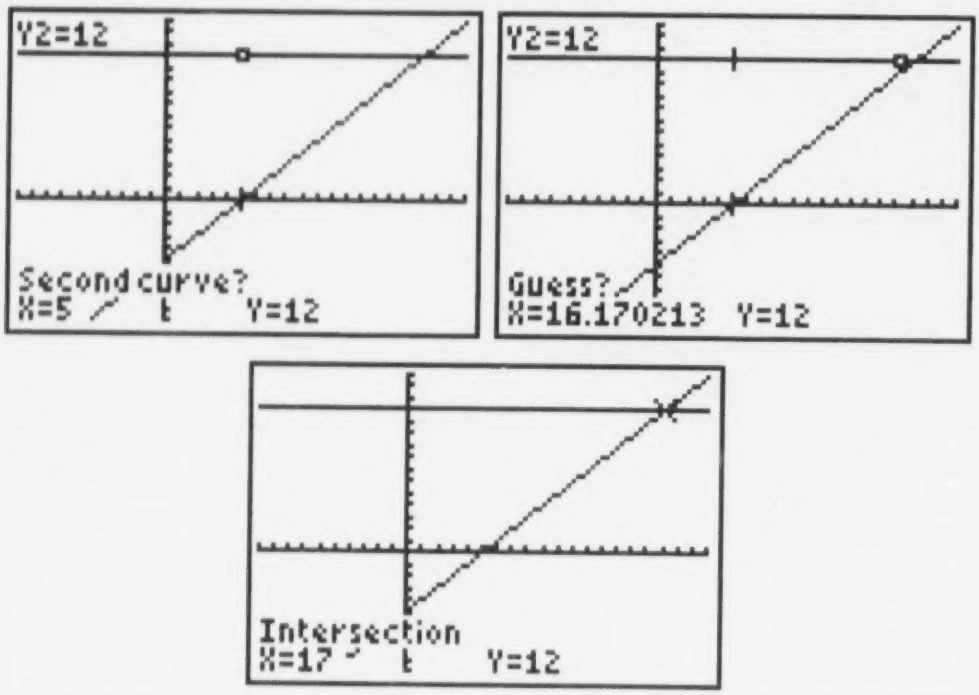




- Find the point of intersection of the two lines by pressing <**2nd**> <**TRACE**> (Calc) <**5**> (Intersect) and then <**ENTER**>, <**ENTER**>, <**ENTER**>. Note: For Guess? have the cursor near the point of intersection.





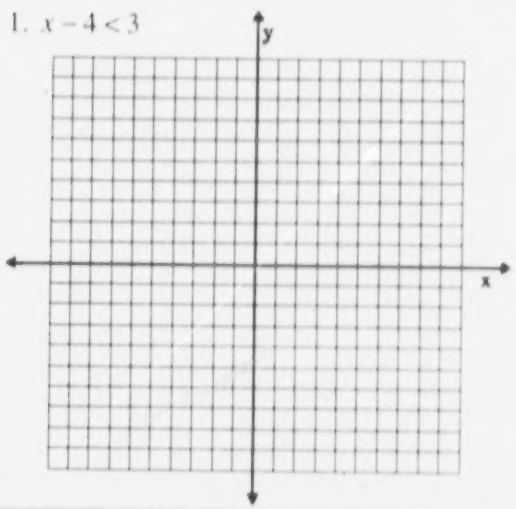


- Observe that the graph of $y = x - 5$ is below (less than) that of the graph of $y = 12$ for all values of x to the left of (less than) 17. Therefore, the solution is $x < 17$.

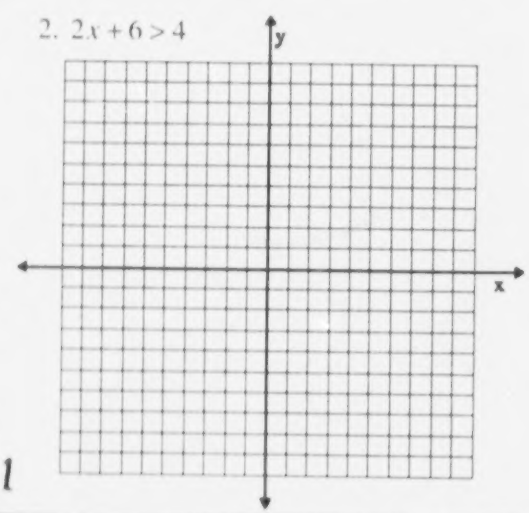
Excercises

a. Graph each of the following on your calculator, then sketch the graph on the grid provided.
b. Indicate the solution set with a statement of inequality.

1. $x - 4 < 3$



2. $2x + 6 > 4$

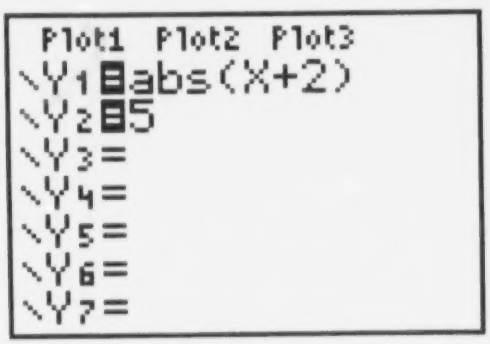


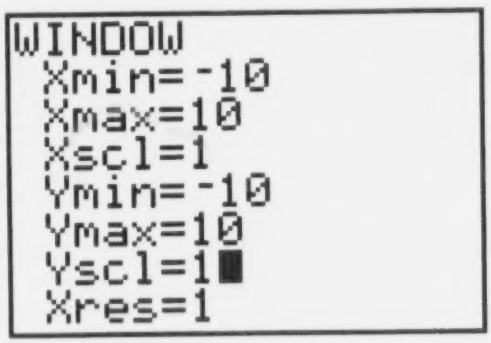
Activity Two Handout: Absolute Value Inequalities

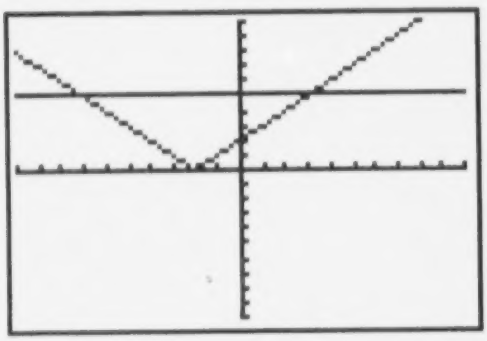
Example:

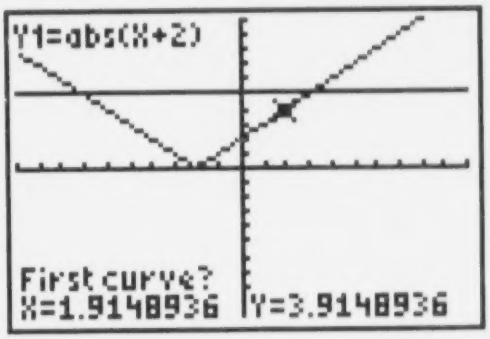
$|x + 2| > 5$

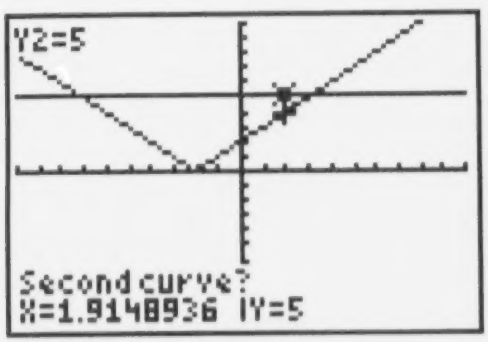
- Again, use the graphing technique. ***Note:*** abs(is found at MATH, NUM, 1:.

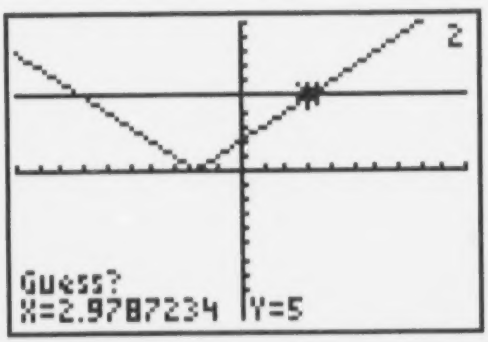


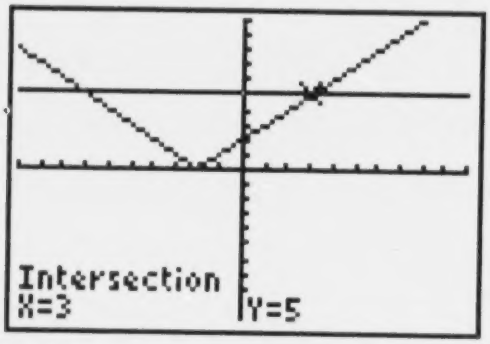




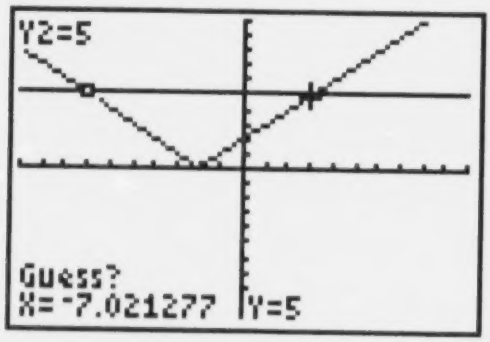


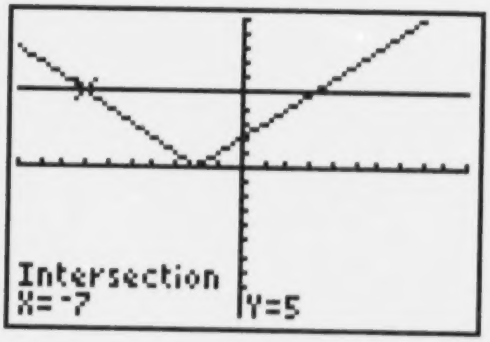






- Repeat the above procedure.





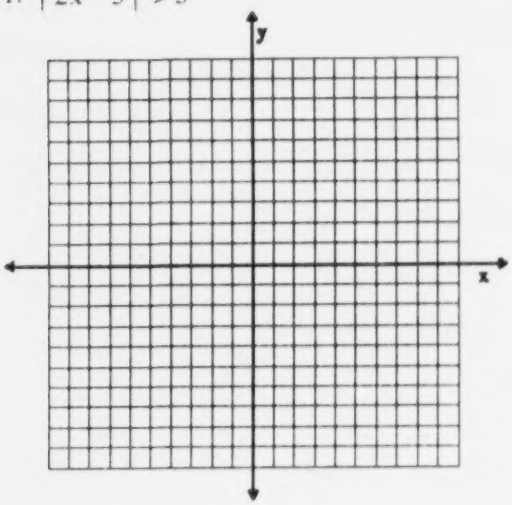
Note: The calculation for intersection must be applied *twice* in order to find both points. The first time move the cursor close to one intersection point for Guess?; the second time, move the cursor over to the other point of intersection for Guess? before pressing the third **<ENTER>**.

- We may write our solution to the inequality by noting that the absolute value graph has values greater than the line $y = 5$ when it is above it. That occurs for values of x that are to the left of -7 or to the right of 3. Thus the solution is the union of the two sets, $x < -7$ or $x > 3$.

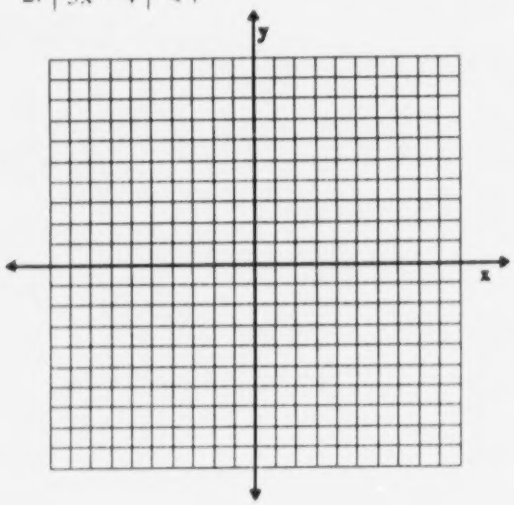
Exercises:

a. Graph each of the following on your calculator, then sketch the graph on the grid provided.
b. Indicate the solution set with a statement of inequality.

1. $|2x-3|>5$



2. $|3x-4|<7$

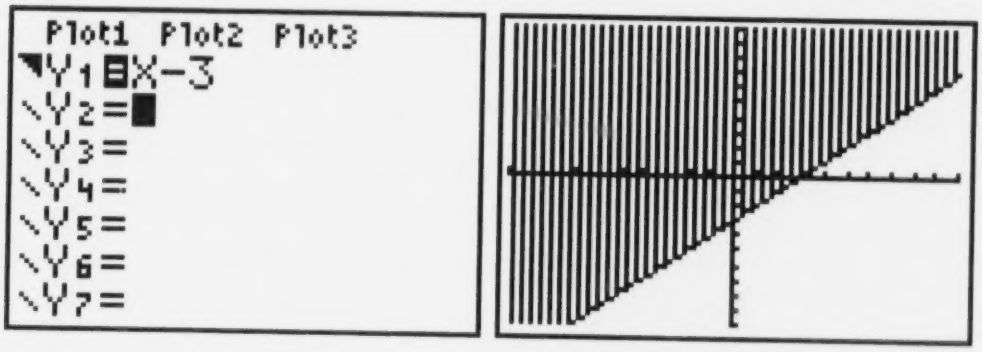


Activity Three Handout: Graphing Inequalities in Two Variables

Example:

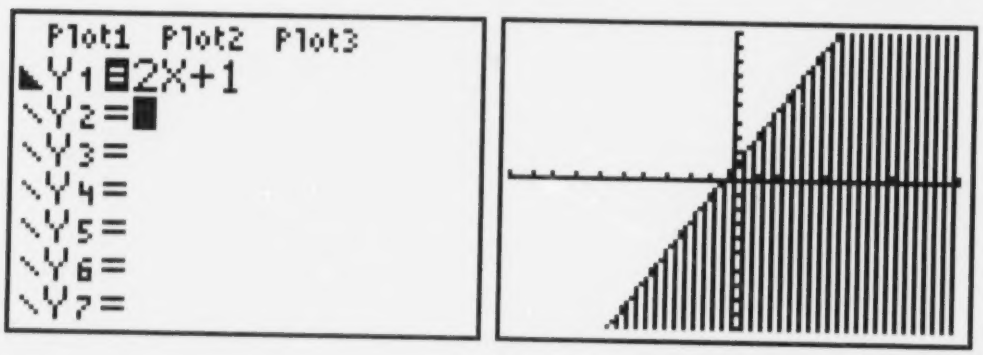
$y \geq x - 3$

- Entering $x - 3$ in Y_1 will graph the line $y = x - 3$. To graph the inequality, we need all points above the line where $y = x - 3$ as well as those on the line. To accomplish this, use **<◄>** to move the cursor to the left of Y_1. Press **<ENTER>** until shading is shown above the slanted line to the left of Y_1. (See left screen below.) Pressing **<GRAPH>** in an appropriate window gives the desired solution.



Note: The graph of $y > x - 3$ must be graphed in the same manner. When transferring the graph to paper, the difference is noted by a solid line when equality is included; a broken or dashed line when it is not.

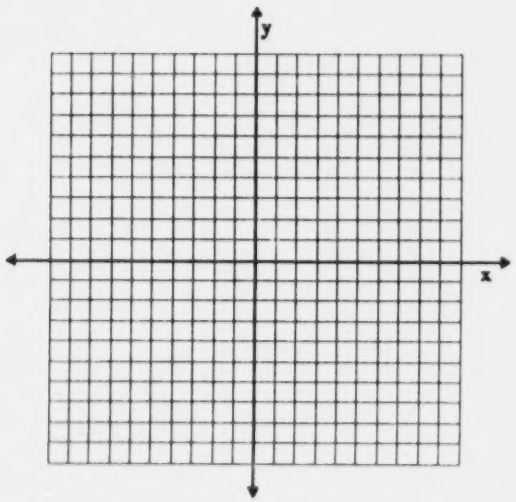
- For $y < 2x + 1$, the area below the slanted line on the **<Y=>** screen should be shaded.



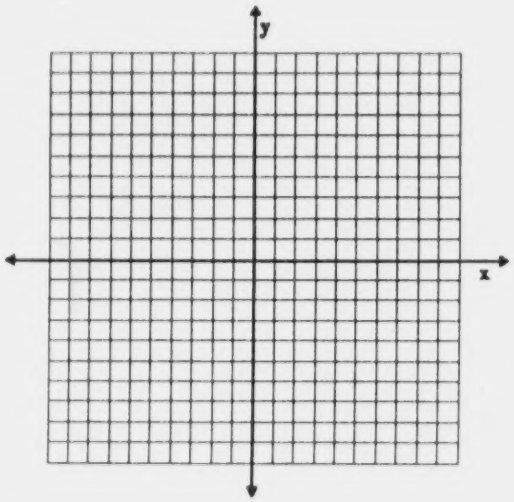
Exercises

a. Graph each of the following on your calculator, then sketch the graph on the grid provided.
b. Indicate the solution set with a shaded region on the graph.

1. $y < 3x - 5$



2. $2x + y > 4$



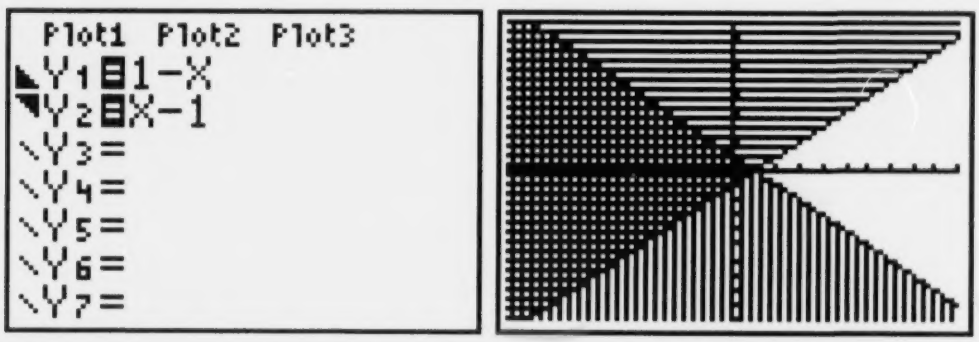
Activity Four Handout: Compound Inequalities

Example:

$x + y < 1$

$y \geq x - 1$

- First solve for y in $x + y < 1$: $y < 1 - x$.
- Enter these in the <**Y=**> menu including appropriate shading as in Activity Three.



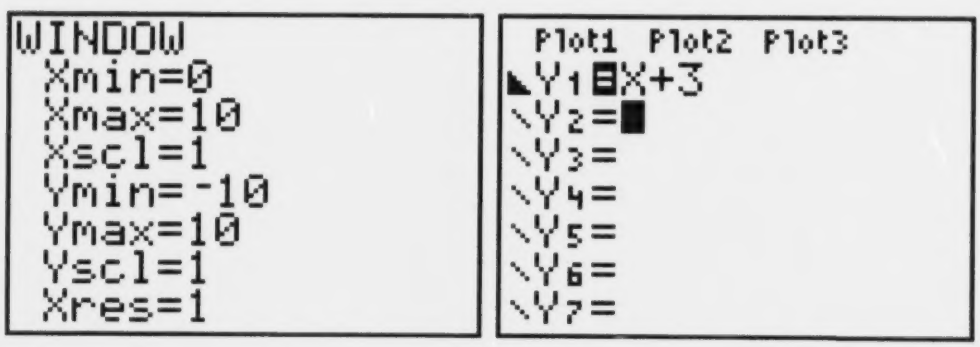
The area where the shading overlaps is the solution set.

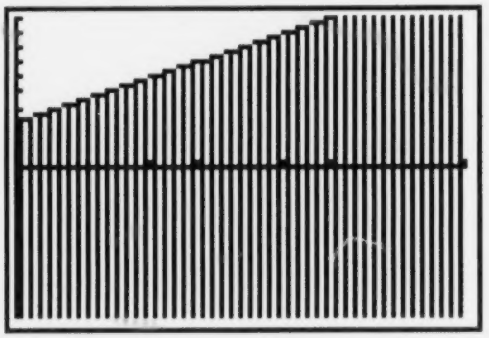
- For inequalities involving x as the only variable, the <**WINDOW**> is the best method of isolating the given area.

Example:

$x > 0$

$y < x + 3$

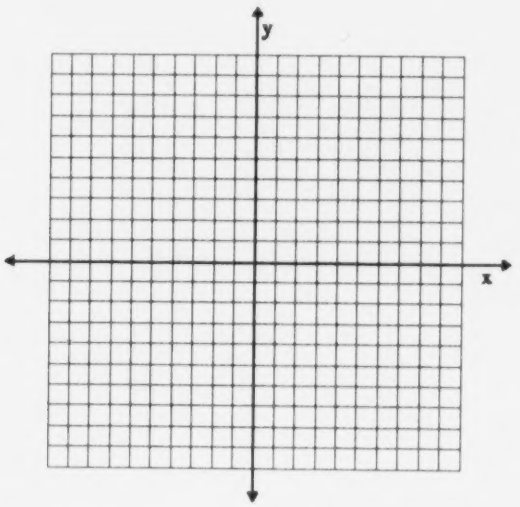




Exercises

- Graph the following system of inequalities on the calculator, and sketch it on the grid provided.

$x \leq 3$
$y < 6$
$y \geq -x + 1$



- Is the given point a solution to this system of inequalities? Justify each algebraically as well as graphically.

(1,3)_____

(4,1) _____

(-5,6) _____

Lesson 4: Starting With Stats

Authors

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Approximate Time Needed For Lesson:
Two traditional class periods

Content Area
Statistics and Probability

Process Standards
Problem Solving, Reasoning, Connections, Communication

Goals for Students

- To collect and organize data from various sources
- To find measures of central tendency
- To choose, construct, and analyze the appropriate graphical representation for a data set using both calculator and non-calculator methods

Description or Summary of Lesson

The intended audience would include students in middle grades (7-8), Pre-Algebra, Math Tech I, Algebra I, and/or Probability and Statistics. Students will work individually or in small groups. The students will collect real-world data for one-variable statistical data analysis using both non-calculator and calculator activities.

Background Information

Students should have basic keystroke skills on a TI-82 or TI-83 and have studied basic statistics prior to this lesson.

Materials and Equipment

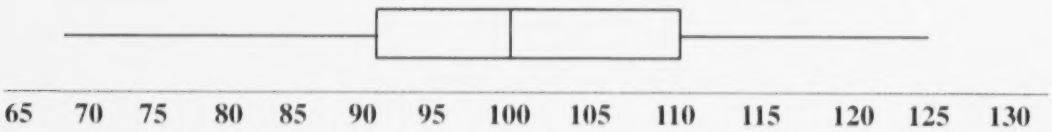
- View screen with graphing calculator
- Overhead projector
- One TI-82 or TI-83 for every two students
- Resources for data: almanacs, Internet (such as the variety of government statistics found at <http://www.lib.utk.edu/gpo/fedstats.html>), magazines, encyclopedias, *Book of Facts*, etc.
- Each student will need:
  - √ graph paper
  - √ handouts

Activity One: Non-Calculator Analysis

Teacher Notes

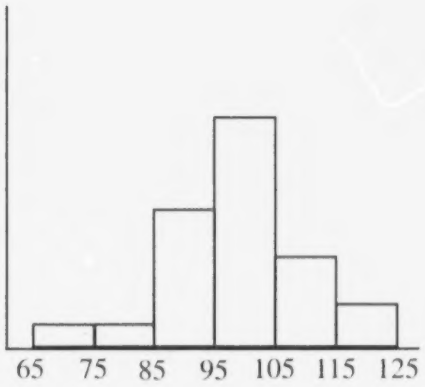
- Divide students into pairs to complete Activity One Handout.

1. Mean IQ is sum of the 25 numbers divided by 25. 99.36
2. Data must be put in numerical order to find the median. After arranging the data in ascending (or descending order) choose the middle, or 13th, number. 100
3. Mode is the data item that occurs most often in the set. 100
4. 68
5. 125
6. Range = 125 - 68 = 57
7. Quartiles divide the data into 4 groups. Since the median is the 13th item in the set, the lower quartile occurs between the 6th and 7th numbers and the upper quartile between the 19th and 20th numbers. Lower quartile = (89 + 92)/2 = 90.5
8. Upper quartile = (110 + 110)/2 = 110
9. Place points above the number line at the lower extreme (68), lower quartile (90.5), median (100), upper quartile (110), and upper extreme (125). Draw a box that has 90.5 and 110 as the endpoints. Draw a vertical segment in the box at 100. Draw a segment from the end of the box at 90.5 to 68 to form the left whisker and another segment from the end of the box at 110 to 125 to form the right whisker.



10. Inter quartile range = upper quartile - lower quartile = 110 - 90.5 = 19.5
11. Teacher will need to discuss with students what an outlier is. Outliers can be calculated by multiplying the inter quartile range (IQR) by 1.5. Add the result to the upper quartile and subtract it from the lower quartile. If any number in the data is outside the interval made by these results, it is an outlier. 1.5 (IQR) = 1.5 (19.5) = 29.25
    90.5 - 29.25 = 61.25 No number in the data is less than this.
    110 + 29.25 = 139.25 No number in the data is more than this. There are no outliers.
12. Choose 10-unit intervals of width, such as 65-75, where the interval includes the lower limit but must be less than the upper; e.g., greater than or equal to 65 but less than 75. Record the frequency in a table and draw the histogram. (See next page.)

Interval	Frequency
65-75	1
75-85	1
85-95	6
95-105	10
105-115	4
115-125	2



13. Before the writing assignment you might want to discuss normal distribution, middle half of data being in the inter quartiles (the box in the box and whisker plot), length of whiskers representing the "spread" of the data, etc. After each student writes his analysis, switch partners and discuss the findings.

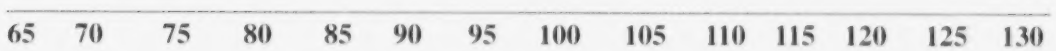
Activity One Handout

- IQ's of 25 ninth-grade students from Anytown, USA:

89	102	92	100	110
111	117	95	85	84
101	110	68	121	125
97	85	103	97	100
88	93	99	112	100

- Answer the following about the 25 Anytown students:

1. What is the mean IQ?_____
2. What is the median IQ?_____
3. What is the mode of the IQ's? _____
4. What is the lowest IQ?_____
5. What is the highest IQ?_____
6. What is the range of IQ's?_____
7. What is the Lower Quartile value?_____
8. What is the Upper Quartile value?_____
9. Draw a box and whisker plot for this data using the number line below.



10. What is the inter quartile range?_____
11. Are there any outliers? _____
12. On your graph paper, construct a frequency table and histogram for the IQ data.
13. Write a paragraph to explain the analysis of the data. Switch partners and compare your findings with those of your new student partner.

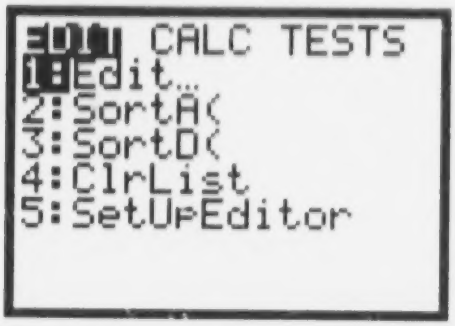
Activity Two: TI-82 or TI-83 Calculator Analysis

Teacher Notes

The teacher will guide the students through this exercise using the Activity Two Handout. This activity is designed to prepare the students to analyze the data they collect in Activity Three.

Activity Two Handout

1. Enter the data from Activity 1 into your calculator by pressing **<STAT>** **<1>** (Edit).



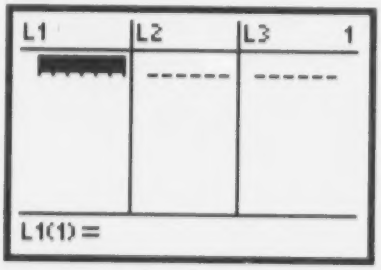
TI-83

TI-82

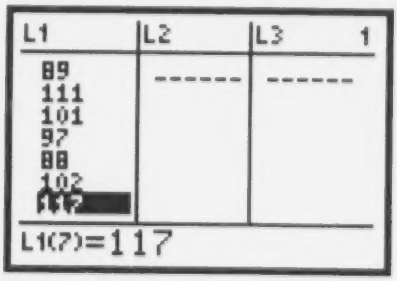
2. If the lists have data already entered in the columns, use the up arrow to highlight L1, press the **<CLEAR>** key and the **<ENTER>** key. This should clear the data from that column. You may clear each of the columns this way.

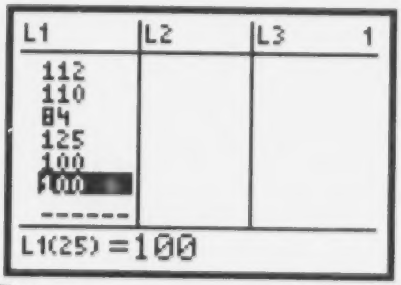
3. You may also go back to the screen above by pressing **<STAT>** and **<4>** followed by the lists you want to clear separated by commas and then **<ENTER>**. L_1 is above the 1 key (**<2nd>** **<1>**), L_2 above the 2 key, etc. This will clear all the lists at once.

4. When the lists have been cleared, press **<STAT>** **<1>** (Edit). The displays for TI-82 and TI-83 are similar.

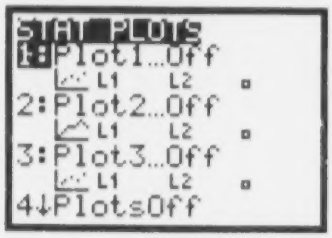


5. Enter data by typing 89, press **<ENTER>**; 111, **<ENTER>**; 101, **<ENTER>**, etc., until all the data has been entered.

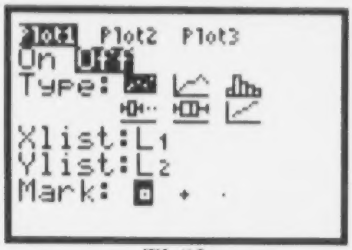




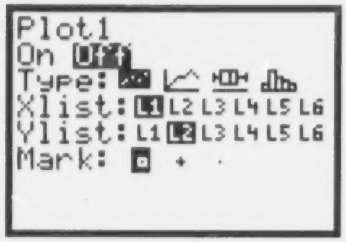
1. Construct a box and whisker plot for the data by pressing <**2nd**> <**Y =**> (STAT PLOT).



2. Choose 1: by pressing <**ENTER**>



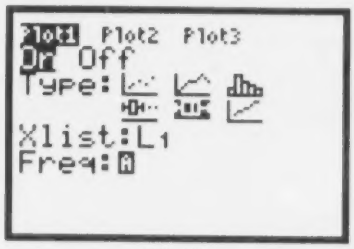
TI-83



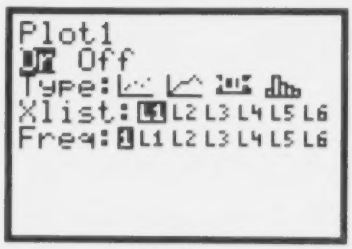
TI-82

3. Press <**ENTER**> to turn the plot on. <▼> one time and then <▶> to the box and whisker plot. The TI-83 will have 6 choices for Type—choose the 5th one. The TI-82 will have four choices—choose the 3rd one. Press <**ENTER**>.

4. <▼> one time. If Xlist is not set at L_1, do this by pressing <**2nd**> <**1**>. The Frequency should be set at 1.

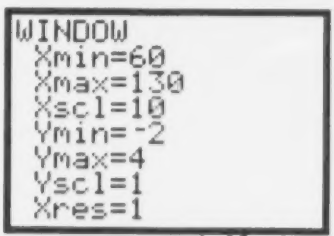


TI-83

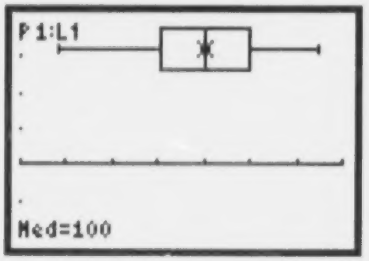


TI-82

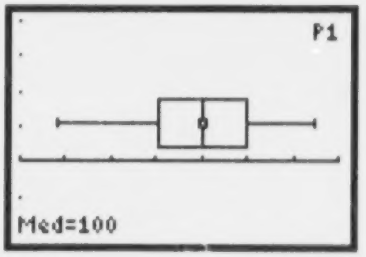
5. Set the viewing window by pressing <**WINDOW**> and making the following entries: Xmin = 60, Xmax= 130, Xscl = 10, Ymin = -2, Ymax = 4, Yscl = 1



6. Press <**GRAPH**> to see the box and whisker plot. Press <**TRACE**>.



TI-83

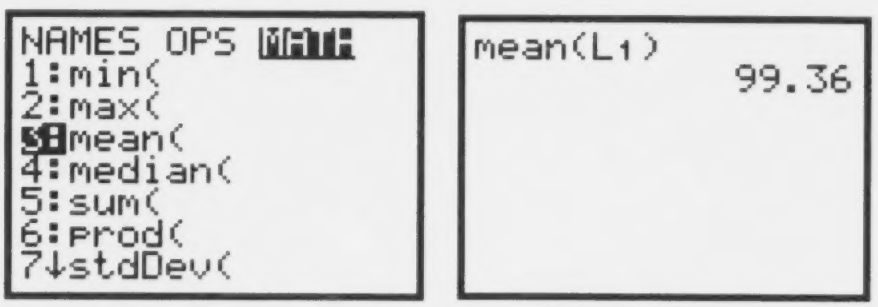


TI-82

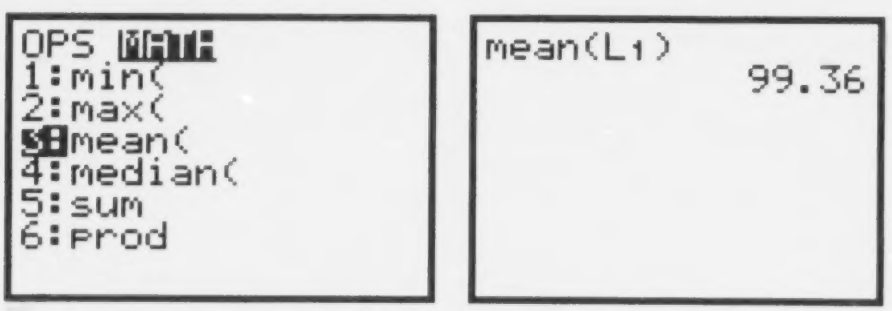
7. Notice the <**TRACE**> key is showing the median IQ. Press <▶> once to see the Upper Quartile(Q3) and press again to see the Upper Extreme. By pressing the <◀> you can find the Lower Quartile (Q1) and the Lower Extreme.

8. To find the mean IQ using the calculator press <**2nd**> <**STAT**> and then press <▶> to highlight MATH. Choose 3: (mean). Press <**ENTER**>.

9. Mean will appear on the home screen. Press <**2nd**> <**1**> and then <) > to name L1 as the argument of the function. Press <**ENTER**> and the answer appears.



TI-83



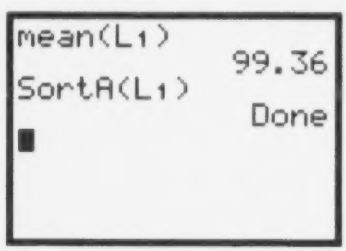
TI-82

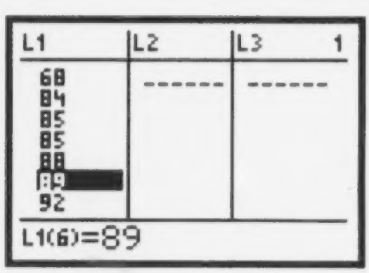
10. To find the mode of the data, the data should be placed in numerical order. This can be done by pressing <**STAT**> and choosing 2 SortA(.

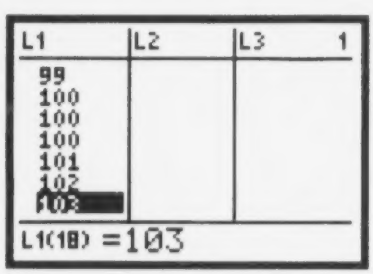
11. Press <**ENTER**> and press <**2nd**> <**1**> <) > to enter L_1 as the argument.

12. Press <**ENTER**> to arrange the data in numerical order.

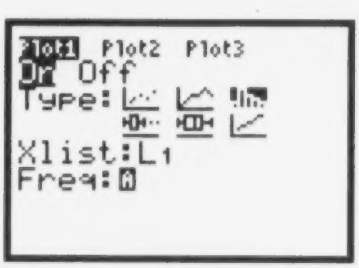
13. Press <**STAT**> and choose Edit, press <**ENTER**> to see the sorted data. Choose the mode from the sorted data.



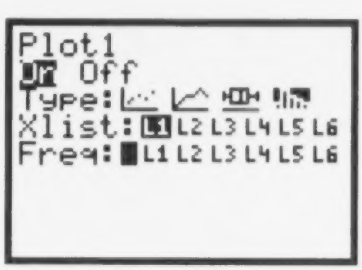




14. To see the histogram return to the STAT PLOT menu by pressing <**2nd**> <**Y =**>. Change the Type from the box and whisker plot to the histogram.

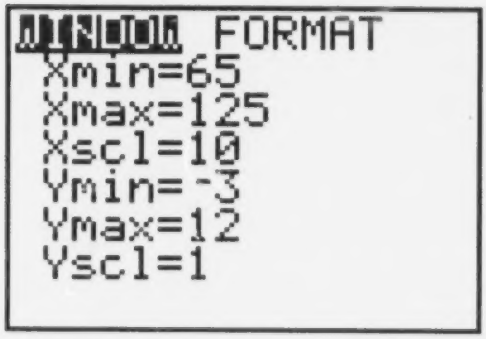


TI-83



TI-82

15. Press <**WINDOW**> and set Xmin = 65, Xmax = 125, Xscl = 10, Ymin = -3, Ymax = 12, and Yscl = 1. Press <**GRAPH**> to see the histogram.



16. You will want to do exploration using the <**TRACE**> key.

Activity Three: Analysis of Real-World Data

Teacher Notes

Outline:

- Topics should be discussed with class and selection made to avoid duplication of data. Some suggestions for topics would be salaries of tennis players, height of team members on a professional basketball team, weight of team members from school football program, maximum speeds of animals, number of consonants in names of class members, grades on quiz, or similar one-variable data.

- Students should be given 2-3 days to collect data. Any library source, Internet source, or personal collection of data should be allowed.

- This activity would be be done as a project.

- Assessment should be based on originality of data, completion of project on time, neatness, organization of data, and quality of analysis.

Activity Three Handout

1. Choose a topic to research. _____

2. Collect data that has at least 10 items. Sources for data research:
   - √ Magazines, especially sports topics
   - √ Almanacs, e.g., *Information Please Almanac, The World Almanac*
   - √ *Book of Facts*
   - √ Encyclopedias
   - √ Internet sources, e. g., <http://www.lib.utk.edu/gpo/fedstats.html> has a wealth of a variety of government statistics including Census data, economic, agricultural, etc.

3. Source of Data (title of publication or URL for Internet data)

   _____

4. Using Activity Two as a guide, analyze the data. If a box and whisker plot or histogram does not best represent your data, try some other graphing representation. Sketch a graph of your data below and answer the following questions:

   - What is the mean for your data ?_____
   - What is the median of your data? _____
   - What is the mode?_____
   - What is the lowest _____?_____
   - What is the highest _____?_____
   - What is the range of_____?_____
   - What is the Lower Quartile value?_____
   - What is the Upper Quartile value?_____
   - Are there any outliers? _____

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5. Write a paragraph to explain the analysis of the data.

Adaptations:

- For upper-level mathematics courses, include longer data sets, standard deviation, variability, frequency distribution, normal distribution with probability.
- Other types of graphical representation such as stem and leaf plots or scatter plots and prediction equations could be included.
- Comparisons of two sets of data with two box and whisker plots or stem and leaf plots could be made.

Suggested Teacher Resource:

Investigating Probability and Statistics, Using the TI-82 Graphics Calculator by Graham A. Jones, Beverly S. Rich, Carol A Thornton, and Roger Day, Addison-Wesley Publishing Company, 1996. (Student Edition and Teacher Guide)

Lesson 5: Investigating Slope

Author

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Approximate Time Needed for Lesson

Two traditional class periods or one 90-minute class period

Content Area

Patterns, Functions, and Algebraic Thinking; Spatial Sense and Geometric Concepts

Process Standards

Connections, Problem Solving, and Communication

Goals for Students

- To graph linear functions
- To determine the slope of a line given any two points on the line
- To articulate the relationship between the calculated slope of a line and its equation
- To compare the slopes of parallel lines and perpendicular lines
- To determine sets of parallel or perpendicular lines from their equations
- To use slope as rate of change to interpret and construct graphs in real-world situations

Description or Summary of Lesson

Students in Pre-Algebra, Algebra I, or Algebra II are the intended audience. In the first activity, students will work individually or in pairs to explore the relationship between slope and linear equations and to compare the slopes of parallel or perpendicular lines. In the second activity students apply the concept of slope to describe a real-world situation. The graphing calculator will be used to enhance the investigations.

Background Information

This is a short, exploratory lesson that connects algebra and geometry and the real world. It could serve as an introduction to a study of function families and "head of family" functions. Prior to this lesson, students should be able to solve a linear equation for y in terms of x and to use the formula to calculate slope. They should also be able to do coordinate graphing and have familiarity with basic keystrokes on a graphing calculator.

Materials and Equipment

- Graphing calculator with view acreen, if available
- Overhead projector
- Each student will need:
  - √ access to a graphing calculator
  - √ 1–2 sheets of graph paper
  - √ red and black markers
  - √ handouts

Activity One

Teacher Notes

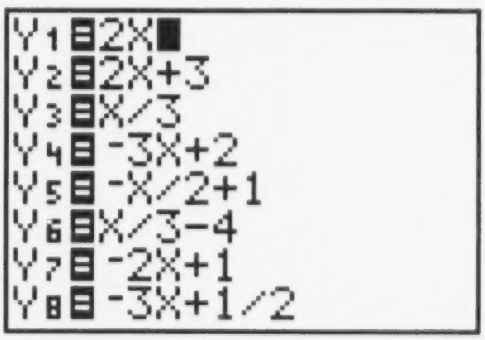
Outline:

- Review solving equations for y in terms of *x* and the slope formula.
- Review geometric concepts: parallel lines and perpendicular lines
- Students complete Activity One Handout

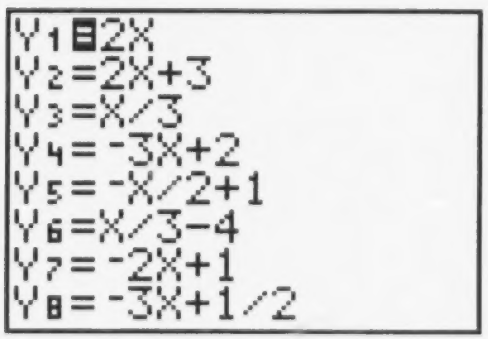
Instructions for the Graphing Calculator:

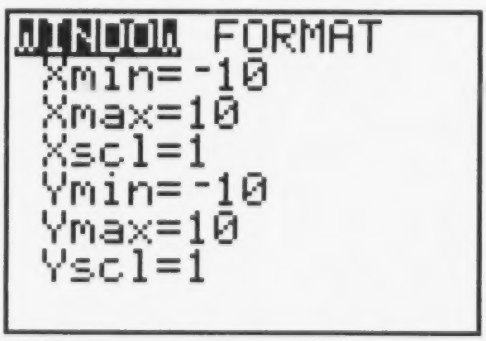
Note: Press <**2nd**> <**Y=**> (STAT PLOT) <**4**> <**ENTER**> to turn all the stat plots off before beginning this activity.

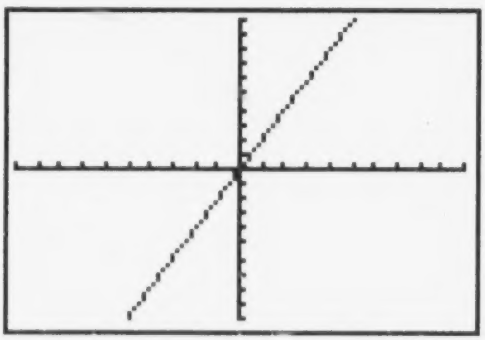
- After algebraically solving an equation for y in terms of x, enter an equation into the calculator, by pressing the <**Y=**> key and the keyboard to enter each equation.



- Graph one equation at a time for this activity to be most effective. To deactivate an equation, use the arrow key and move the cursor onto the = sign in the <**Y=**> page. Press <**ENTER**>. The = key should no longer be highlighted. Set an appropriate window by pressing < **ZOOM**> <**6**>. The graph will appear on the screen. (See below.)

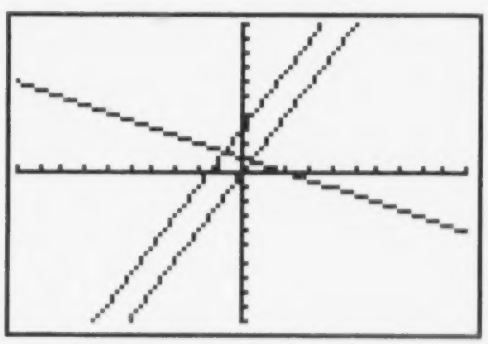




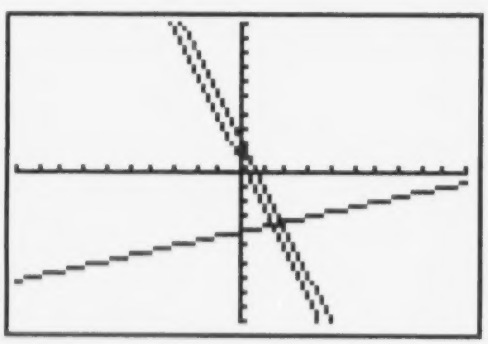


- Repeat the above procedure for the other seven equations.

- To graph more than one equation, use the arrow key and the enter key to activate the needed equations. For example, it is interesting to graph Y_1, Y_2, and Y_5 at the same time or perhaps Y_4, Y_6, and Y_8.

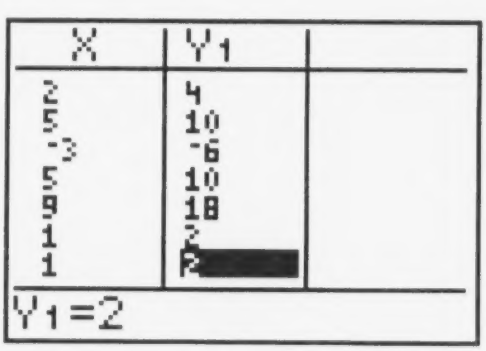


Y_1, Y_2, and Y_5



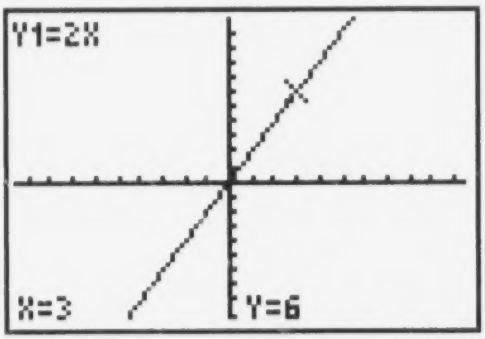
Y_4, Y_6, and Y_8

- Find coordinates for two points by pressing <**2nd**> <**Graph**> (TABLE) and entering any two values for x in the X. The corresponding y-coordinates will apprear in the Y column (On the TI-83 press <**ENTER**> in the Y column for each coordinate to appear after entering an x.)



Note: <**TRACE**> and <**▶**> will provide coordinates of points on the graph, but the values may be unnecessarily difficult to use.

- Or find the coordinates for two points by using **<2nd>** **<TRACE>** (CALC) **<1>** (value). Enter an *X* value, press **<ENTER>**, and the corresponding *Y* value will be calculated for you.



TI-83

Note: The equation of the line shows in the upper left corner of the TI-83 screen.

- Use the calculator to calculate the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ and the two points from TABLE or from **<2nd>** **<TRACE>** **<1>**.

Activity 1 Handout

1–8. Solve each equation in the following chart for y in terms of x. Graph each equation using the graphing calculator. Using <**2nd**> <**TRACE**> (CALC) <**1**> (value), find the coordinates for 2 points on each line. Using the graphing calculator and the slope formula, calculate the slope of each line. Complete the chart and use the information to answer the questions that follow.

	Functions	$y =$	Coefficient of x	2 Ordered Pairs	Calculated Slope
1	$\frac{y}{2} = x$				
2	$-2x + y = 3$				
3	$3y = x$				
4	$y + 3x = 2$				
5	$2y = -x + 2$				
6	$3y = x - 12$				
7	$y + 2x = 1$				
8	$2y + 6x - 1 = 0$				

9. What do you notice about the coefficients of x and the calculated slope?

10. How do the slopes of functions 1, 2, and 5 compare?

11. Graph the three functions simultaneously and sketch the graphs below. How are the graphs of the functions related?

12. Is there any significance among functions 3, 4, 6, and 8?

13. Predict what their graphs look like and sketch below, labeling each line. Check your prediction by graphing the functions simultaneously.

14. In the chart below solve each equation for y in terms of x. Use what you learned above to determine the slope for the graph of each function.

	Functions	$y =$	Slope
1	$8y = 16x - 32$		
2	$2y = -x + 8$		
3	$6x - 3y = 18$		
4	$6y = -3x + 24$		
5	$2y = -4x + 10$		

15. List any pairs of lines that you think are parallel or perpendicular. Check your assumptions by using the graphing calculator.

16. Write a paragraph describing your conclusions from this investigation.

Activity Two

Teacher Notes

Outline:

- Review how to calculate slope. Students complete #1-#6 on the Activity Two Handout.
- Students use graph paper to plot the ordered pairs at specific points of the path of a roller coaster.
- Students complete Activity Two Handout.
- Teacher and students discuss what appears to be happening between various points on the curve. (Lead the student to discover positive slope, negative slope, same slope, zero slope, etc.)
- Students enter data into calculator to verify their graph.
- Students calculate the slopes to test their assumptions.

Answers for Activity Two Handout:

Part I.

1. 1/2
2. -2
3. 1
4. 1/2
5. Yes, #1 and #4 have the same slope.
6. No, they could be collinear. However, from the graphs, they are parallel.

Part II.

1. Positive Slope

   Example: (0,0) and (20,5)

   Slope: 1/4

2. Negative Slope

   Example: (120,130) and (140,120)

   Slope = -1/2

3. Zero Slope

   Example: (220,48) & (240,48)

   Slope = 0

4. Same Slopes

   Example: (50,40) & (60,60) (300,120) & (280,80)

   Slope = 2 Slope = 2

   Significance: This indicates that the two segments could either be parallel or collinear. Look at graph to determine which.

5. Opposite Slopes

   Example: (40,20) & (50,40) Example: (150,100) & (160,80)

   Slope = 2 Slope = -2

   Significance: This indicates that on one of the sections the roller coaster is rising at the same rate that it is falling on the other section.

Extension:

- Have students create their own roller coaster such that it has some horizontal sections, some sections that seem to have the same slope, etc.

Activity Two Handout

Part I.

Find the slope of the line segments connecting each pair of points below:

1. (0,4)
   (4,6)

2. (3,-5)
   (5,-9)

3. (3,6)
   (6,9)

4. (8,6)
   (12,8)

5. Do any of the line segments have the same slope?

6. If you answered yes in #5, does this mean that the segments connecting the points are parallel? To test your assumption plot the line segments below. Hint: Put a sheet of graph paper behind this handout to help with your graphs.

Part II. Graphing the path of the roller coaster

Problem: Graph the ordered pairs below on a sheet of graph paper with an appropriate scale. For a more realistic roller coaster graph, connect the points using smooth curves instead of jagged line segments. Assume the roller coaster starts at (0,0).

(0,0) (20,5) (40,20) (50,40) (60,60) (70,80) (80,100) (100,120)

(120,130) (140,120) (150,100) (160,80) (180,62.5) (200,47)

(220,48) (240,48) (260,58) (280,80) (300,120)

1. Mark with a heavier black marker sections of the graph where the roller coaster is going up.

2. Mark with a red marker sections of the graph where the roller coaster is going down.

3. Compute the slopes between the pairs of points for the black sections.

4. Compute the slopes between the pairs of points for the red sections.

5. What do you notice about the slopes in #3 and #4?

6. Are there any segments where the roller coaster appears to be traveling horizontally? Calculate the slope of any such segments.

7-8. For #7 and #8, pick pairs of segments that appear to fit the following conditions. Calculate the slope to test your assumptions.

7. Same slope (What is the significance of having the same slope?)

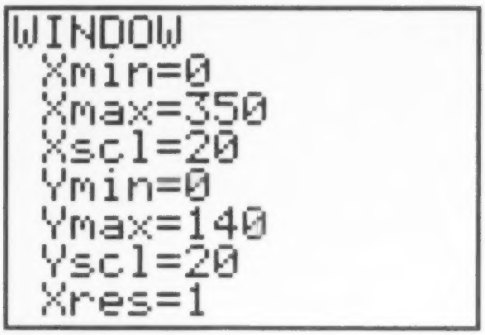
8. Slopes that are opposite (What is the significance of having an opposite slope?)

9. Enter the data into the Graphing Calculator TI-82 or TI-83:

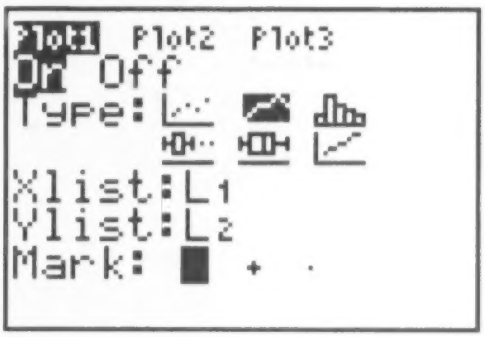
- Press **<STAT> <1>** (Edit).
- TI-82: Enter the x-coordinates into the L_1 column and the y-coordinates into the L_2 column. The screen should appear like the following screen shot.

L1	L2	L3 5
0	0	------
20	5	
40	20	
50	40	
60	60	
70	80	
80	100	
L3 =		

- Set an appropriate window such as the one below.



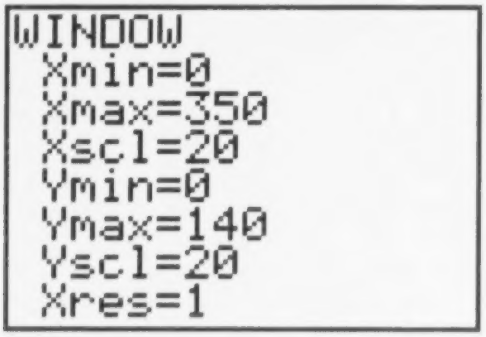
- Press <**2nd**> <**Y=**> (STAT PLOT) and <**1**>.



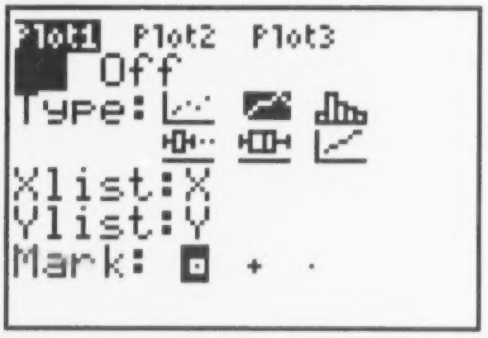
- To enter L_1 for Xlist, press the <**2nd**> <**1**> (L_1). To enter L_2 for YList, press the <**2nd**> <**2**> (L_2).
- TI-83: Enter the *x*-coordinates into the *X* column and the *y*-coordinates into the Y column.
- The screen should appear like the following.

X	Y	Y2 3
0	0	------
20	5	
40	20	
50	40	
60	60	
70	80	
80	100	
L2 =		

- Set an appropriate window such as the one below.

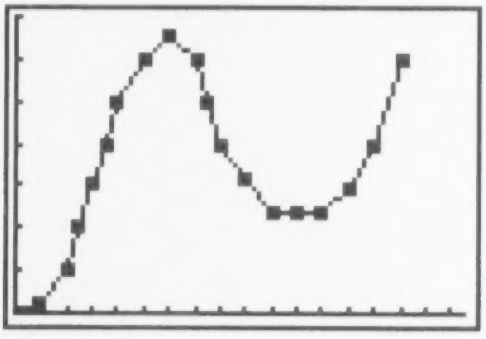


- Press <**2nd**> <**Y=**> (STAT PLOT) and <**1**>.



- To enter X for Xlist, press the <**ALPHA**> <**STO▶**> (X). To enter Y for YList, press <**ALPHA**> <**1**> (Y).

- TI-82 or TI-83: Press the <**Y=**> key and either delete or deactivate the functions. Now press <**GRAPH**>. The graph should appear like the one below.



- Note that the coordinates of each of the data points along the track can be seen by pressing <**TRACE**>.

Lesson 6: Connecting Measurement

Authors

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Approximate Time Needed for Lesson

One 90-minute class period or two traditional class periods

Content Area

Estimation, Measurement, and Computation; Statistics and Probability; Spatial Sense and Geometric Concepts; Patterns, Functions, and Algebraic Concepts

Process Standards

Connections, Problem Solving, and Reasoning

Goals for Students

- To collect and summarize data
- To describe data with measures of central tendency and dispersion such as mean, median, quartiles, and extremes
- To use formulas to derive measurements for volume and surface area in problem-solving situations
- To measure to the nearest inch and quarter inch
- To express measurements in different ways
- To use trial and error and successive approximations to derive missing values in a formula

Description or Summary of Lesson

Students in middle grades, Math Tech I, Pre-Algebra, or Algebra I are the intended audience. This lesson includes activities that connect measurement to geometry, statistics, estimation, and the real world. Students may work individually or in pairs.

Background Information

Prior to this lesson students should have had experience with measuring length; box and whisker plots; measures of dispersion and central tendency including mean, median, quartiles, and extremes; and formulas for area and volume of simple geometric figures. Some prior experience with calculators would be helpful but is not required.

Materials and Equipment

- View screen with TI-82 or TI-83 graphing calculator
- Overhead projector, if available
- Assortment of cylindrical and rectangular solids
- Rulers in inches and/or centimeters
- Each pair of students will need:
  - √ access to a TI-83 or TI-83 graphing calculator
  - √ handouts

Activity One: Connecting Measurement to Statistics and Surface Area

Teacher Notes

Outline:

- Teacher discusses non-standard and standard units of measurement and history of some of our common units of measurement.
- Students complete the data collection and use a graphing calculator to describe the data as detailed in Activity One, Handout 1.
- Students convert cubits to feet and apply a formula for surface area of a rectangular solid in the problem-solving situation described in Activity One; Handout 2.
- Students convert cubits to feet and apply a formula for surface area of a rectangular solid in the problem-solving situation described in Activity One, Handout 3.

Traditionally measurements for ancient structures were in cubits, a measurement of length equal to the length from the elbow to the fingertips. A cubit, however, would vary from individual to individual. Determine the length the class might use as a measurement for the cubit.

Note: Although the dictionary says a cubit is sometimes represented by 18 inches or as much as 22 inches, it is more interesting to use a measure of central tendency such as the mean or median to describe or define or "standardize" a cubit.

- Each student measures from his or her elbow to the tip of the middle finger. The information is recorded in the individual's journal and also on the chalkboard or a chart. Each student or pair of students enters the data from the board into the graphing calculator as a list (as detailed in Activity One, Handout 1) and develops a box and whisker plot using the TI-82 or TI-83 in order to describe the data with the mean, median, quartiles, and extremes.
- Each student will discuss with his or her partner or neighbor whether the median or the mean would be the most appropriate estimation of cubit. The teacher will discuss the findings and class consensus for the size of a cubit in contemporary units.

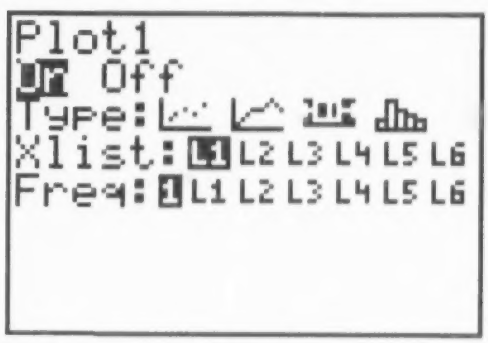
Note: When completing Handout 2, 2*L*W should be omitted since the roof and floor would not be painted. When completing Handout 3, be sure students use the calculator keys for π (<**2nd**> <^> on the TI-82 and TI-83).

- The area of each Norman window would be $8 + \frac{1}{2}\pi \times r^2$ sq. ft., which is approximately 9.57 sq. ft. since the radius is 1.

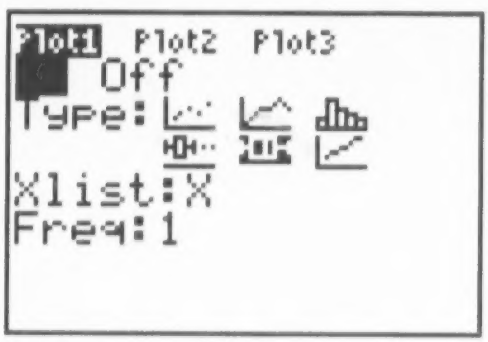
Activity One, Handout 1

Use a TI-82 or TI-83 to graph the class measurements for a cubit, and describe a cubit as the mean or median of the data collected. Enter the data for the class into your graphing calculator as follows.

- Press <**STAT**> <**1**>. If numbers are already in the first list (L_1 or X for example) clear by highlighting the heading and pressing <**CLEAR**> and <**ENTER**>.
- < ▼ >. Enter information in the first list (L_1 or X) by entering each student's measurement for a cubit then press <**ENTER**> until all the data is in the list.
- Next press <**Y=**> and either clear each Y or make sure no = sign is highlighted.
- Then press <**2nd**> <**Y=**> (STAT PLOT) <**1**>.
- Select ON by using <**ENTER**> and < ▼ > (Type).
  TI-82: <▶> to the third graph (box and whisker) and press <**ENTER**>.
  TI-83: <▶> to the fifth graph (box and whisker) and press <**ENTER**>.
- <▼> to Xlist and over to the list that contains your info (see screens below), <**ENTER**>. Leave or set Freq: 1.



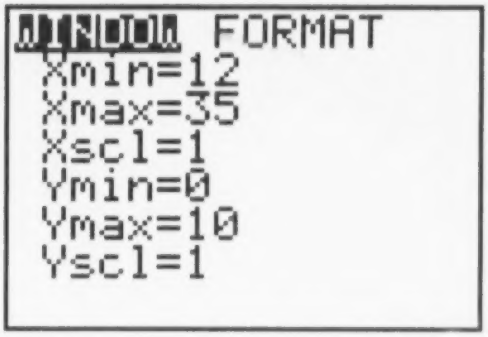
TI-82



TI-83

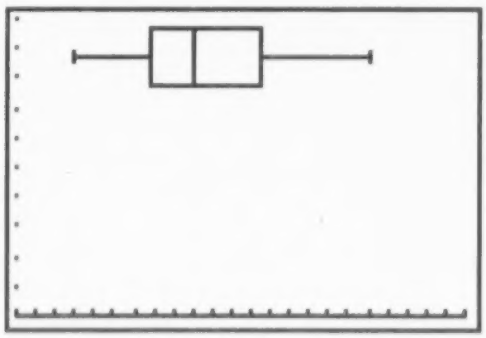
- Set an appropriate window by pressing <**WINDOW**> so that Xmax and Ymax will incorporate your data. See the following sample windows:

TI-82

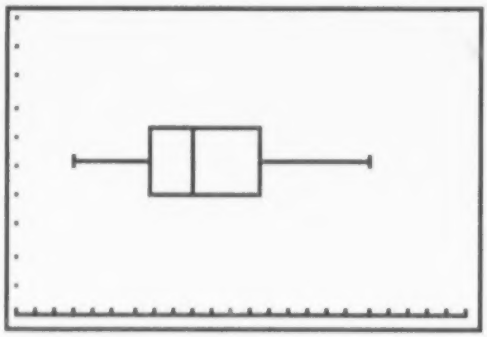


TI-83

- To see the box and whiskers graph of the class data, press <**GRAPH**>.



TI-82



TI-83

- Sketch your graph below.

- Press <**TRACE**> to move the cursor along the plot to obtain data such as the median, extremes, and quartiles.

- To find the mean, press <**2nd**> <**STAT**> (LIST) and
  TI-82: <▶> (Math).
  TI-83: <◀> (Math).

- Then press <**3**> (mean( ).

- Enter the name of your list using the keyboard. For example, to find the mean of L_1 on the TI-82, press <**2nd**> <**1**> (L_1) and <)> <**ENTER**>.

Median _____ Mean _____

Q1 _____ Q3_____

Min X_____ Max X_____

- Do you think the mean or the median best describes a cubit? Why?

_____

_____

_____

Activity One, Handout 2

Problem:

A four-year institution in Dayton, TN, built its administrative wing using the dimensions of an ancient boat. Suppose this vessel were 300 cubits long, 50 cubits wide, and 30 cubits tall. Using the class consensus for cubit from Handout One, convert these dimensions to feet.

- Use these conversions to calculate the surface area of the building. Recall that surface area for a rectangular solid is 2(LW) + 2 (WH) + 2(LH). However, do not paint the roof or the floor of the building. Disregarding windows, determine how much paint would be needed to paint the building if a gallon of paint covers 350 square feet.

Hint: You can use your calculator to store the values needed to find the surface area by omitting the part of the formula that applies to the roof and the flooring. Store values by using the following procedure.

TI-82 Enter length in feet and press **<STO►> <ALPHA> <L> <2nd> < , > (:).**
Enter width in feet and press **<STO►> <ALPHA> <W> <2nd> < , > (:).**
Enter height in feet and press **<STO►> <ALPHA> <H> <2nd> < , > (:).**

TI-83 Enter length in feet and press **<STO►> <ALPHA> <L> <ALPHA> < , > (:).**
Enter width in feet and press **<STO►> <ALPHA> <W> <ALPHA> < , > (:).**
Enter height in feet and press **<STO►> <ALPHA> <H> <ALPHA> < , > (:).**

- Then use similar keys as needed to enter the portion of the formula that you will be using and complete the calculation for the total square feet to paint. From this answer calculate the gallons of paint.

Formula you used: _____

Total square feet: _____

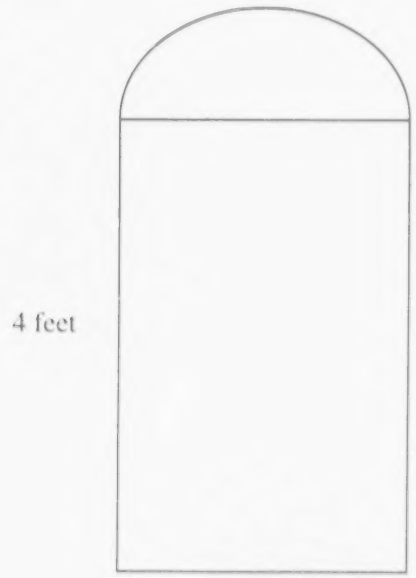
Gallons of paint needed: _____

Activity One, Handout 3

Problem:

- If we suppose the administration building from Handout 2 has 40 Norman windows such as the one pictured below, use your graphing calculator to find the amount of paint needed.

  Hint: Use keys similar to the ones you used in Handout 2.



Area of each window: _____

Total area of windows: _____

Total area to paint: _____

Number of gallons of paint needed: _____

Activity Two:
Connecting Measurement to Estimation and Volume

Teacher Notes

Outline:

- Teacher reviews formulas for finding the volume of rectangular prisms and cylinders.
- Working in pairs, students complete Handout 1.
- Teacher reviews trial-and-error method of problem solving and method of successive approximations.
- Working in pairs, students complete Handout 2.

- A variety of rectangular prisms and cylinders are needed for this activity. The teacher may ask the students to collect and bring to class during the two or three weeks before the activity.

- For any of the calculations involving pi, students should use **<2nd>** **<^>** (π) rather than the approximation 3.14.

- The linear measurements may be done in inches or in centimeters. Cubic centimeters or milliliters may be easier to estimate than cubic inches. Students may need encouragement to estimate the volumes.

- Handout 2 key for missing values to two decimal places; the teacher may want to have the estimates just to one decimal place or to the nearest unit.

Cylinder	Radius	Height	Calculated Volume
1	12 units	1.04 units	470 cubic units
2	4.9 units	13 units	980 cubic units
3	71 units	4.75 units	75,224 cubic units
4	20 units	5 units	6283 cubic units

Activity Two, Handout 1

Directions:

- Complete the chart below for three rectangular prisms. Use the calculator to find the volume for the first prism. **Before** finding the volume of the second prism, write an estimate based on the calculated volume of the first prism. **Before** calculating the volume of the third prism, write an estimate based on the calculated volume of the first two.

Rectangular Prism	Length	Width	Height	Estimated Volume	Calculated Volume
1					
2					
3					

- Complete the chart below for three cylinders. Use the calculator to find the volume for the first cylinder. **Before** finding the volume of the second cylinder write an estimate based on the calculated volume of the first prism. **Before** calculating the volume of the third cylinder, write an estimate based on the calculated volume of the first two.

Cylinder	Radius	Height	Estimated Volume	Calculated Volume
1				
2				
3				

Activity Two, Handout 2

Directions:

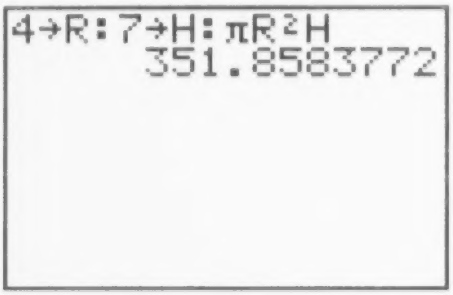
- Use the graphing calculator to store the volume calculation for a cylinder with R=4 and H=7.

TI-82 Enter <**4**> <**STO►**> <**ALPHA**> <**X**> (R) <**2nd**> <**.**> (:).
Enter <**7**> <**STO►**> <**ALPHA**> <**^**> (H) <**2nd**> <**.**> (:).

Press <**2nd**> <**^**> (π) <**ALPHA**> <**X**> (R) <x^2> <**ALPHA**> <**^**> (H) <**ENTER**>.

TI-83 Enter <**4**> <**STO►**> <**ALPHA**> <**X**> (**R**) <**ALPHA**> <**.**> (:).
Enter <**7**> <**STO►**> <**ALPHA**> <**^**> (H) <**ALPHA**> <**.**> (:).

Press <**2nd**> <**^**> (π) <**ALPHA**> <**X**> (**R**) <x^2> <**ALPHA**> <**H**> <**ENTER**>.



- Now the formula is stored and it can be repeated for other radii and heights by pressing <**2nd**> <**ENTER**> (ENTRY). Then by pressing <**2nd**> < ◄ >, the cursor will move to the beginning of the line. Replace the values of R and H to check some of your calculations made on Activity Two, Handout 1. To change the values, you can use the <**2nd**> <**INS**> and/or <**DEL**> keys.

- For this activity complete the chart by assigning the given values and using trial and error and successive approximations to estimate the missing values. Hint: Use the values above to help determine the first value to use in the trial-and-error method.

Cylinder	Radius	Height	Calculated Volume
1	12 units		470 cubic units
2		13 units	980 cubic units
3	71		75,224 cubic units
4		5 units	6283 cubic units

Lesson 7: Constructing Trigonometric Concepts

Author

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Approximate Time Needed for Lesson

Three traditional class periods or two 90-minute blocks with students doing some of the work and constructions as homework

Content Area

Spatial Sense and Geometric Concepts; and Patterns, Functions, and Algebraic Thinking

Process Standards

Connections, Problem Solving, and Communication

Goals for Students

- To apply geometric concepts such as straight edge and compass constructions, coordinate geometry and theorems to develop an understanding of basic trigonometric functions such as the sine and cosine

Description or Summary of Lesson

The intended audience is students in Geometry, Algebra II, or Pre-Algebra classes who have been exposed to theorems about chords of circles, perpendicular line segments, and basic straight edge and compass constructions. The lesson is a good introduction to sine and cosine graphs using connections to previously studied math concepts. Working in pairs, students will complete an investigation that leads to the concepts of the basic trigonometric ratios and functions. The graphing calculator is used to complete the investigation.

Background Information

In addition to basic geometric constructions and theorems, students should have prior experience with basic keystrokes and making scatter plots on a graphing calculator.

Materials and Equipment

- View screen with graphing calculator and overhead projector, if available
- Each pair of students will need:
  - √ access to a graphing calculator
  - √ straight edge, compass, and protractor
  - √ fairly large circular object such as garbage can lid, plate, round pan, frisbee, tire cover, etc.
  - √ piece of string as long as the diameter of the circular object
  - √ large sheet of heavy paper or poster board
  - √ well-sharpened pencil
  - √ handout

Activity

Teacher Notes

Outline:

- ✏ Students collect circular objects that are too large to trace on a sheet of notebook paper but not too large for a sheet of poster board.
- ✏ Teacher reviews straight edge and compass construction of perpendicular lines.
- ✏ Students are divided into pairs, and each pair is given poster paper, a protractor, a straight edge, and a compass.
- ✏ Students complete the measuring and constructing described in Handout 1: Student Instructions.
- ✏ Using graphing calculators, students answer the questions in Handout 2.
- ✏ Students share findings.
- ✏ Teacher and students summarize findings.
- ✏ Students complete Handout 3, *optional*.

Note: Some innovative students may wish to tape together several sheets and make a much larger circle. This is fine; they will just have to be more careful when they take their measurements.

- **The center of their circle is best obtained by drawing two chords on their circle, constructing the perpendicular bisector of each chord, and finding the intersection point. A theorem from Geometry states that the perpendicular bisector of a chord of a circle will pass through the center of the circle.**

- Encourage students to make their own measuring sticks rather than using inch or centimeter rulers. They need to be careful that the divisions they make on the stick are carefully drawn, but the fact that they will arrive at the graphs of the sine and cosine curves from using arbitrary measuring units is central to this activity.

- When they divide the circumference of their circle into 36 equal segments, they need to make the marks on the circle fairly accurate. Extending the lines on a protractor is one way to do this. Dividing the circumference by 36 and using this as a measuring unit will also work. Wrapping a string about the object may be helpful in measuring the circumference.

- The teacher may need to show the students how to do elementary scatter plots on their calculator, and some may need help in correctly entering their data into lists. Enter the numbers {0,10,20,30,...,350} into the first list. (If you have a TI-83, you may wish to use the capability of your calculator to actually give a name to your list. You may wish to name the list *DEG*.)

- Have the students take the measurements that determine the X and Y coordinates carefully. These are the most critical measurements in the activity.

- **The X values are recorded in L_2, and the Y coordinates in L_3. If you have a TI-83, these lists may be named *X* and *Y*.**
- Here is an example of what the edit screens may look like:

L1	L2	L3 1
0	15	0
10	14.772	2.6047
20	14.095	5.1303
30	12.99	7.5
40	11.491	9.6418
50	9.6418	11.491
60	7.5	12.99

L1 ={0, 10, 20, 30, ...

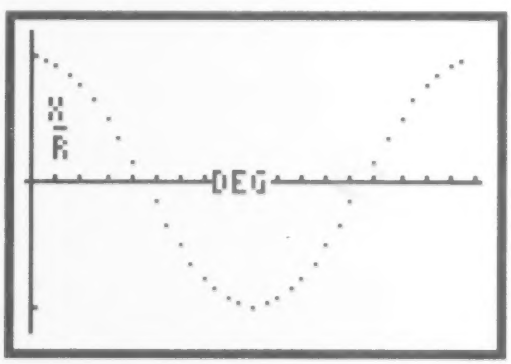
TI-82 or TI-83

DEG	X	Y 3
0	15	0
10	14.772	2.6047
20	14.095	5.1303
30	12.99	7.5
40	11.491	9.6418
50	9.6418	11.491
60	7.5	12.99

Y(1) =0

TI-83 with Named Lists

- If we imagine a unit length named the farka, and a circle with a radius of 15 farkas, then we will get a set of values that can be used as a check on the student's values.
- *(These were generated by using the standard Sin and Cos functions, but they give an idea of what some student values may look like if done correctly.)*
- After highlighting L_4 and entering L_2 (or list *X*) divided by the radius (using the new units), a new set of numbers appears in L_4. After highlighting L_5 and entering L_3 (list *Y*) divided by the radius, a new set of numbers appears in this list.
- An ideal scatter plot of L_4 *(X/R)* vs L_1 *(DEG)* would look similar to this:



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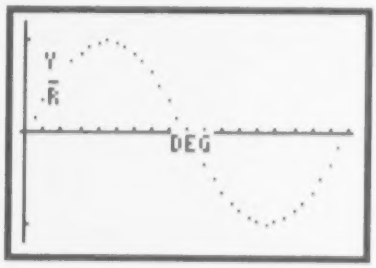
- An ideal scatter plot of L_5 *(Y/R)* vs L_1 *(DEG)* would look like this:



Selected Answers and Comments for Handout One, Student Instructions:

3. Perpendicular bisectors of two chords of a circle will intersect at the center of the circle.

5. The center of the circle is the midpoint for each diameter.

6. Quadrants should be labeled in the standard way, with I, II, III, IV listed in a counter-clockwise direction.

7. This will obviously depend on what value that the students choose for their unit length.

8. 10 degrees

9. 36 elements, representing angles in degrees, spaced at 10-degree intervals. 360 degrees is not needed since it is located in the same position as the 0 degree mark.

11. The dimension of each list is 36

- The questions at the end of the assignment ask students to recognize that the sin and cos graphs represent ratios of numbers, and that even though different groups had different sizes of circles, and measured with different measuring scales, the ratios of the *X* and *Y* coordinates to the radius of their particular circle were the same as the ratios for all of the other circles. In all circles, the ratio of the circumference to the diameter is a constant, namely, pi.

Suggested Extensions:

1. It is productive to have the groups repeat the activity using standard meter or yard sticks. They should be encouraged to talk about why they will get the same graphs, no matter what measuring standard they used to take measurements.

2. More advanced classes can explore the reciprocal trig functions (csc, sec and cot) by taking the reciprocals of their lists and forming the appropriate scatter plots.

3. Once students recognize that cosine values are obtained from the scatter plot of their List L_2/R, which is now contained in L_3, and that the sine values are contained in list L_5, they can begin to explore some elementary trigonometric identities such as:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

They should use the values in their own lists, or values from their graphs, instead of calculator values or values from tables.

4. Students with a TI-83 can try the SinReg regression option in the Stat Calc menu (#C in the menu).

Suggestions for ASSESSMENT

1. ***Constructions***

   50 points—students should be rewarded for the time and effort they put into gathering the data

   25 points—successfully completing the activity and producing a graph, even if it is wrong

   15 points—geometric constructions using straight edge and compass

   10 points—cooperative work with a partner

   5 points bonus—making an accurate, non-standard measuring stick

2. ***Data collection and analysis***

   50 points—accuracy, as entering data correctly is an important part of the activity

   15 points—accurate measurements of X and Y coordinates

   15 points—correctly entering values in lists and determining how to get the rest of the data in later lists

   15 points—well constructed and correct scatter plots

   5 points—labeling axes and giving a scale on final sketches of graphs

Answers to Handout 2

1. The *X* and *Y* coordinates of a point on a circle must be less than or equal to the length of the radius. Thus the ratios *X/R* and *Y/R* will be $\leq$ 1. The coordinates are + or -, depending on which quadrant the coordinate is located in.

   Thus, the cos and sin functions can be at most +1, and no less than -1.

2. L_4 will be + when the points are in Quadrants I and IV (where the *X* coordinates are + since R is always +).

3. L_5 will be + when the points are located in Quadrants I and II (where the *Y* coordinates are + since R is always +).

4. Draw a smooth curve between the points on the scatter plot and estimate the values for the sin and cos from the *y* coordinates of points on the smooth curve through the scatter plot.

   The sin and cos will have the same value when the angle is 45 degrees, and again when the angle is 225 degrees. The graphs intersect at these points.

5. Estimate these values, **using your graphs**: (all angles are in degrees)
   Values should be approximately:

sin(35) = .57	cos(45) = .71
cos(55) = .57	sin(90) = 1.0
sin(135) = .71	cos(135) = - .71
cos(225) = -.71	sin(225) = -.71
sin(305) = -.82	cos(325) = .82

Essay:

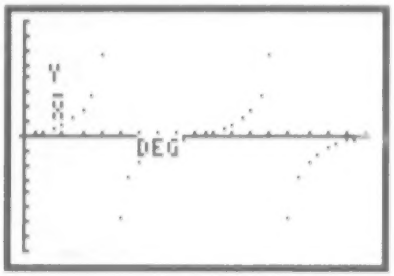
- This is one of the most important parts of the activity, as it asks the students to understand why they are getting the same numbers and graphs from circles and measuring sticks that are so vastly different. Be sure to clearly explain the underlying principle that ratios between coordinate values and the radius of the circle are the same, no matter what size the circle is, or what is used to measure it.

- This may be a difficult idea for students to verbalize. As stated in the earlier comments, **the sin and cos graphs represent ratios of numbers, and even though different groups had circles with different sizes, and measured with different measuring scales, the ratios of the *X* and *Y* coordinates to the radius of their particular circle was the same as the ratios for all of the other circles**. In all circles, the ratio of the circumference to the diameter is a constant, namely, pi.

Answers to Activity Handout 3

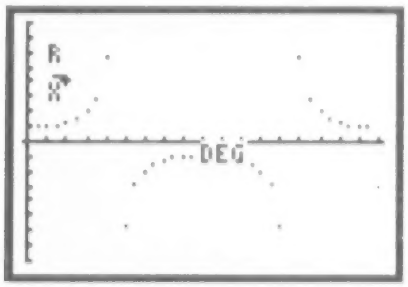
1. Students should be getting a scatter plot of the tangent graph. To get around the problem of division by 0, which is what happens at the 90- and 270-degree angles, tell students to replace 0 with a very small number like .0001. The results, as far as the graph goes, will be about the same.

   Using the previous lists (in terms of our imaginary farkas):

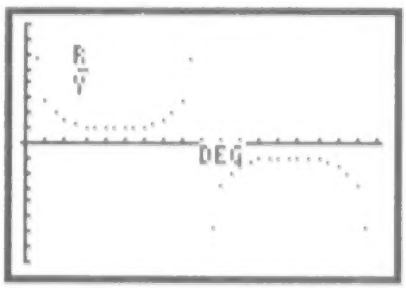


   The graph of L_2 / L_1 will look the same since $tan\theta = \frac{sin\theta}{cos\theta}$

2. The graphs will be the same as the csc() and sec() graphs.



sec ()



csc()

Handout 1: Student Instructions

Note: You will be asked to submit your work, so be sure you retain important measurements and data.

1. Your first task is to bring an object that can be used to trace the outline of a circle. You may not use a compass or a method such as a pencil attached to a length of string. Garbage can lids, frisbees, hoola hoops, round pots and pans, and similar objects work fine. Your object should make a circle too big to fit on a normal sheet of paper, but not so big that it won't fit on a sheet of poster paper.

2. Carefully trace the outline of your object on a sheet of poster paper. Use a pencil for your first outline in case you make a mistake. After you have drawn a good circle, you can trace over your drawing using a pen or fine-tipped marker.

3. Using a straight edge and a compass, find the center of your circle. Draw your lines lightly, as you will be taking measurements from your circle a bit later in this activity. (Hint: what theorems do you know about chords of circles that may help you?)

4. Draw a diameter of your circle.

5. Construct another diameter for your circle, perpendicular to the one you drew in #4. The two diameters intersect at the center of the circle, which is also the _____ for each of the diameters.

6. Extend the two diameters beyond the circle. In effect, you have constructed two coordinate axes. Label one of them the X axis, and the other the Y axis, and decide where the positive X axis is located. Lightly label each of the four quadrants (I, II, III, IV) on your poster paper.

7. Use a measuring stick if you must, but it may be more interesting to take a straight edge (be sure it is longer than the radius of your circle) and make a set of equally spaced marks on it. In effect, you are making a measuring stick with a new set of units. Make up some name for your units. For example, let one unit be 1 farka. You'll now be measuring lengths in terms of farkas, instead of inches or centimeters. Of course, you can use inches or centimeters if you wish.

   What is the radius of your circle in terms of your new unit?

   _____

   What is the equation of your circle in terms of your new unit?

   _____

8. Decide on a method that will divide the circumference of your circle into 36 equally spaced segments (called *arcs*). You may use a protractor and your new measuring stick to do this.

   What is the measure of each of the central angles that have been formed by these arcs? _____. Mark each of these dividing points.

9. Press **<STAT> <1>** (Edit) on your calculator. Enter the numbers {0,10,20,30,...,350} into the first list. (If you have a TI-83, you may wish to use the capability of your calculator to actually give a name to your list. You may wish to name the list *DEG*.)

   How many numbers will you enter into this list? _____

   What do you think that these numbers represent?

   _____

   Why was 360 not entered into the list?

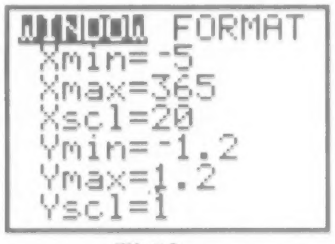
   _____

10. Determine the *X* and *Y* coordinates of each point you marked in #8 in terms of your new units. This may take a while, and can become tedious, but be as accurate as you can, as this is one of the most crucial aspects of this activity. Record the X values in L_2, and the Y coordinates in L_3. If you have a TI-83, you may wish to name these lists *X* and *Y*.

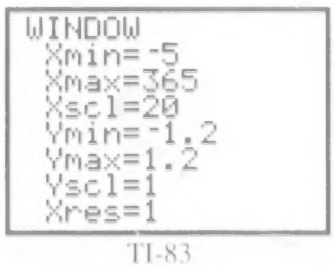
11. The number of elements in each list is called the *dimension* of the list. Check to see that each list has the same dimension.

12. Highlight L_4, and enter L_2 (or list *X*) divided by your radius (use your new units). You should see a new set of numbers appear in L_4. Highlight L_5, and enter L_3 (list *Y*) divided by your radius. You should see a different set of numbers appear in this list.

13. You are now ready to construct a scatter plot of your data. In the **<2nd> <Y=>** (STAT PLOT) menu, turn on Plot 1, and at Type select scatter plot. In the Xlist enter L_1 (list *DEG*), and in the Ylist enter L_2 (list *X*). Choose the dot as the *mark*.



TI-82



TI-83

14. Set your window as follows:

15. Turn off any equations that you may have on the <**Y=** >screen. (You can highlight the =, and press <**ENTER**> to turn off the graphs of these functions without having to get rid of them entirely.)

16. Press <**GRAPH**> to view your scatter plot. Sketch your graph, indicating units on both sets of axes. If you have access to a Graph Link, you can get a printout of your graph.

17. Turn off Plot 1, turn on Plot 2, and repeat procedures 13–16, using L_3 (list *Y*) in place of L_2 (list *X*).

18. Turn on both Plot 1 and Plot 2, and describe how the two graphs compare.

19. Compare your lists and scatter plots with another group's. What lists do your groups have in common and which are different? Did your groups have similar-looking graphs?

20. Post your graphs in a place where other groups can compare their graphs to yours.

Note: The first scatter plot you made is called a **cosine** graph, and is the graph of a function that relates the angles you listed in L_1 to the ratio of the X coordinates of your points over the radius of your circle, which are found in L_4. It is usually written as

$$y = cos\theta$$

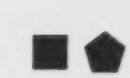
where θ is the angle that you listed in L_1.

The second scatter plot that you made is called a **sine** graph, and is usually written as

$$y = sin\theta.$$

These two graphs and their corresponding functions are two very important concepts in mathematics. You will encounter them often.

Please answer the questions on the accompanying sheet (Handout 2).



Handout 2

Write in complete sentences, using good grammar and correct spelling. Be sure to indicate how you arrived at any of your calculated values.

1. Why were the values in lists L_4 and L_5 between 1 and -1?

   Could they ever be larger than 1, or smaller than -1? Why?

2. In which quadrants were the values in L_4 positive _____, and negative _____? Why was this the case?

3. Answer question #2 for L_5.

4. How could you use your scatter plots to estimate values for the sine and cosine of angles that are not among those listed in L_1 (List DEG)? List any angles for which the sin and the cos of the angle are the same.

5. Estimate these values, **using your graphs**: (all angles are in degrees)

   sin(35) = _____ cos(45) = _____ cos(225) = _____ cos(325) = _____

   cos(55) = _____ sin(90) = _____ sin(305) = _____

   sin(135) = _____ cos(135) = _____ sin(225) = _____

Essay:

- The most amazing thing about this activity is that, even though several of the groups had circles of different sizes, all of the groups arrived at about the same numbers in Lists L_4 and L_5, and they had similar graphs for the sine and cosine curves. This is a very important property of a group of equations known as **trigonometric equations or functions.**
- An equally important factor was that you arrived at the same graphs, even though different groups were using different measuring standards.
- Think about why this is, and write a paragraph or two explaining why you think this happened.

Handout 3: Further Excursions in Trigonometry

1. In L_3 enter the values obtained by dividing L_2 by L_1 (List Y divided by List X). Make a scatter plot with L_1 (the angles) on the X axis, and L_3 on the Y axis. Sketch the resulting graph on another sheet of paper, being sure to state the window values that you used. What happens at the 90-degree and the 270-degree marks? Why does this happen?

   Suppose that you made a scatter plot with L_1 on the X axis, and the list made by calculating the L_5 / L_4 values placed on the Y axis. Compare this graph with the one you made at the beginning of this question. Explain the relationship between the graphs.

2. Explore the following scatter plots. Comment on their most important features. The list L_1 (List DEG) will always be on the X axis of the scatter plot. Make scatter plots with the following lists on the Y axis:

   a. Radius/ L_2 (the reciprocal of L_4, or Radius/List *X*)

   b. Radius/ L_3 (the reciprocal of L_5, or Radius/List *Y*)

Lesson 8: Graph Interpretation

Authors

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Approximate Time Needed for Lesson

Three traditional class periods or two 90-minute periods if all of the activities are selected

Content Area

Probability and Statistics

Process Standards

Communication, Problem Solving, Connections, and Reasoning

Goal for Students

- To describe, extend, interpret, and create a variety of graphs using appropriate materials

Description or Summary of Lesson

Students in middle grades or Pre-Algebra are the intended audience. The lesson is composed of five short activities designed to introduce students to a variety of graphs and possible interpretations in different learning contexts. The activities can be used together or selected ones used in combination with textbook or other graphing activities.

Background Information

This is an introductory lesson that is expandable to more complex mathematics. Prior to this lesson students need only very minimal graphing skills and basic familiarity with a graphing calculator.

Materials and Equipment

- View screen with graphing calculator
- Overhead projector
- Yard sticks
- Newspaper
- Post-it notes
- Each pair of students will need:
  - access to a graphing calculator
  - access to a calculator-based lab (CBL)
  - access to a motion detector
  - access to GraphLink
  - graph paper
  - handouts

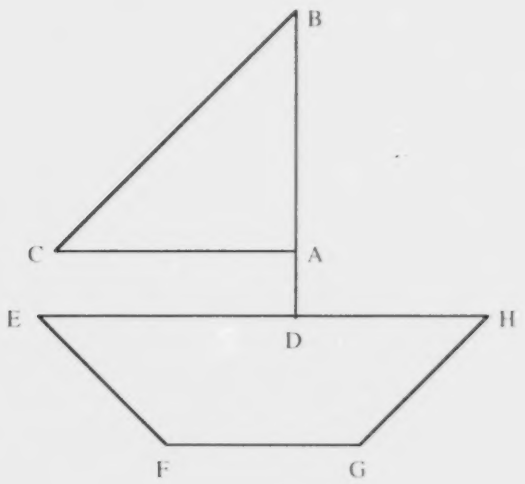
Activity One: Introduction to Coordinate Graphing

Outline:

- ✏ Review north, south, east, and west directions.
- ✏ Give each student a copy of the Activity One Handout and complete Part I.
- ✏ Let students work in pairs to complete Part II.

Part I.

- Give students the following oral directions for locating, labeling, and connecting eight points or dots (A to H) on the handout to form a sailboat:
  - √ Label the beginning point A
  - √ From A go north 15 squares and make a dot; label it B
  - √ Connect point B to point A
  - √ From dot or point B go west 15 squares and south 15 squares and make a dot; label it C
  - √ Connect point B to point C
  - √ Connect point C to point A
  - √ From point A go south 4 squares and make a dot; label it D
  - √ Connect point D to point A
  - √ From point D go west 16 units and make a dot; label it E
  - √ Connect point E to point D
  - √ From point E go south 8 units and east 8 units and make a dot; label it F
  - √ Connect point F to point E
  - √ From point F go east 12 units and make a dot; label it G
  - √ Connect point F to point G
  - √ From point G go east 8 units and north eight units and make a dot; label it H
  - √ Connect point H to point G
  - √ Connect point H to point D



101

- Ask students to relabel the points as ordered pairs where the first member is a number that represents the distance east or west from point A, and the second member is a number that represents the distance north or south from point A. Agree to use positive numbers for east and north and negative numbers for south and west.

A = (0, 0)	B = (0, 15)
C = (-16, 0)	D = (0, -4)
E = (-15, -4)	F = (-8, -12)
G = (4, -12)	H = (12, -4)

Part II.

- Give each pair of students a city map that uses blocks. A Yahoo search on the Internet for city maps will produce a wide variety of maps. For example, the Waterfront and Historic Park in Philadelphia is currently located at the following address: <http://www.libertynet.org/mmp/maps/historic.html>. (Since web addresses occasionally change, a new Internet search for the Waterfront and Historic Park in Philadelphia may need to be conducted.)

- Have students write directions for how to get from one point to another. Ask students to use the directions in the sailboat activity, i.e., E or W and N or S to go to another location on the chosen map. For example, if students are using the map of Philadelphia, they could be asked to travel from the Vietnam War Memorial to the Liberty Bell with as few turns as possible, going E or W first, then N or S. In all cases build understanding of the Cartesian Coordinate system by leading to the (*x*-coordinate, *y*-coordinate) labeling of points.

Suggestions for Assessment:

- Ask students to write or give oral answers to questions such as those below:

1. Suppose I were a pizza delivery driver at the corner of _____Street and _____Street, and I had to make a delivery at the corner of _____Street and _____ Street. Write instructions in terms of E, W, N, S for the best way to go.

- Rewrite your answer as an ordered pair that is equivalent to your written instructions.

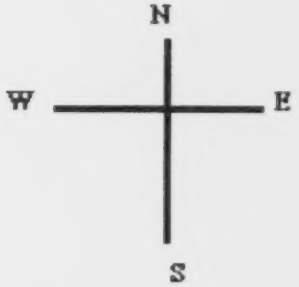
2. Suppose I were a cab driver trying to run up the fare of a passenger I picked up at the corner of _____ Street and _____ Street with a given destination. Write instructions in terms of E, W, N, S for the "best" route.

- Rewrite your answer as an ordered pair that is equivalent to your written instructions.

Extension:

- Have students use the ordered pairs of the segment endpoints in Part I to determine the equation of each corresponding line and then graph on calculator using the <y= > screen. Graph Link could allow printing of the graphs. Students could then outline or highlight the line segments of the picture from Part I.

Activity One Handout



Activity Two: Scatter Plots

Teacher Notes

Outline:

- Introduce the terms "increasing" and "decreasing."
- Students measure and record height (x) in inches and shoe size (y) on Post-it note. Then each student will place his or her Post-it appropriately on a graph drawn on the chalkboard.
- Students record what this graph represents and use descriptive words to describe the relationship. For example, "as the height increases, the shoe size increases" or "taller people tend to have bigger feet."
- Students work in pairs to enter the class's data into a graphing calculator and complete the Activity Two Handout.

Suggestions for Assessment:

- Pairs of students brainstorm several situations that may result in a decreasing graph. Then each person will create a set of data and enter it into the calculator to graph as a scatter plot. Students will then explain in journals the trends in the scatter plot.

Extensions and Adaptations:

- Divide the class according to gender and graph the data again. Is there a noticeable difference? What explanation could you offer for the variations?
- Students may benefit from collecting their own data. For example, have students use the classified section of the newspaper to record information about used car prices of a particular make, but using different models. Construct a scatter plot using the car prices and the age of the cars in list one and list two. Before graphing, remember to change the WINDOW to show values comparable to the prices gathered. Is the resulting plot linear? Increasing? Decreasing? Why are there some points that seem to be out of line with the others? What factors might cause differences in used car prices of the same model? From this information can you determine a line of best fit? Launch into a discussion of slope, which should give students an estimate of how much a car could depreciate from year to year. Based on this calculation, estimate the price of a car for an age that is not listed. Using the same estimate how could the students determine what a new car might cost in 5 years? 10 years?

Activity Two Handout

Technology Activity:

- Using information gathered in your class, create a scatter plot using the TI-82. For example:

Shoe	Height
5	60
8	63
9	68
8	70

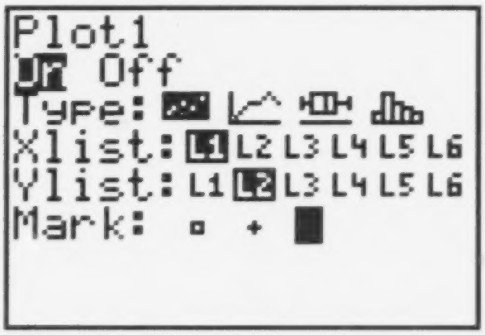
- To enter the data, press **<STAT>** **<1>** (Edit).

Note: If numbers are already in any of the lists, clear by highlighting the heading, i.e., L_1, L_2, etc., by pressing **<CLEAR>**.

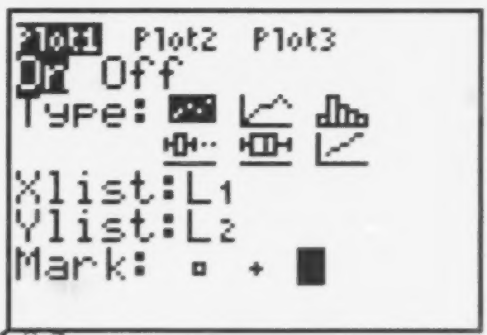
- Then **<Down Arrow>** under L_1 and enter each height. For example,
  <6> **<0>** **<ENTER>**
  <6> **<3>** **<ENTER>**
  <6> **<8>** **<ENTER>**
  <7> **<0>** **<ENTER>**

- **<Right Arrow>** to the next list, L_2, and enter the shoe sizes.
- Press **<Y=>** and delete any information in any of the Ys.
- Then use the following keystrokes to create a scatter plot in the calculator:
  <2nd> **<STAT PLOT>** **<1>** **<ENTER>**
  <Left Arrow> (On) **<ENTER>**
  <Down Arrow> (Type) and highlight the scatter graph.
  Press **<ENTER>**
  Arrow to Xlist and select L_1
  Arrow to Ylist and select L_2

Note: On the TI-83, to enter L_1, for Xlist press **<2nd>** **<1>** (L_1). Similarly, to enter L_2 for the Ylist, press **<2nd>** **<2>** (L_2).



TI-82



TI-83

- Set an appropriate window by pressing <**WINDOW**>. For Xmax, enter a number slightly greater than the tallest person; for Xmin, enter a number slightly smaller than the shortest person. Likewise, change Ymax to slightly more than largest shoe size and Xmin to slightly smaller than the smallest shoe size. Likewise, set appropriate Xscl and Yscl depending upon the ranges for height and shoe size. An appropriate Xscl might be 5 or 10 and an appropriate Yscl might be 1 or 2.

- To see the scatter plot of the data, press <**GRAPH**>. Compare and contrast your graph with the class graph on the board. Discuss what would happen if different values were substituted in the lists.

Activity Three: Graph Interpretation

Teacher Notes

- Encourage students to use the terms "increasing" and "decreasing" in their descriptions of the two problems in the handout. Explain what might create a change represented by each rising, falling, and no-change section for both graphs in the handout. Many possible scenarios would be correct for each one.

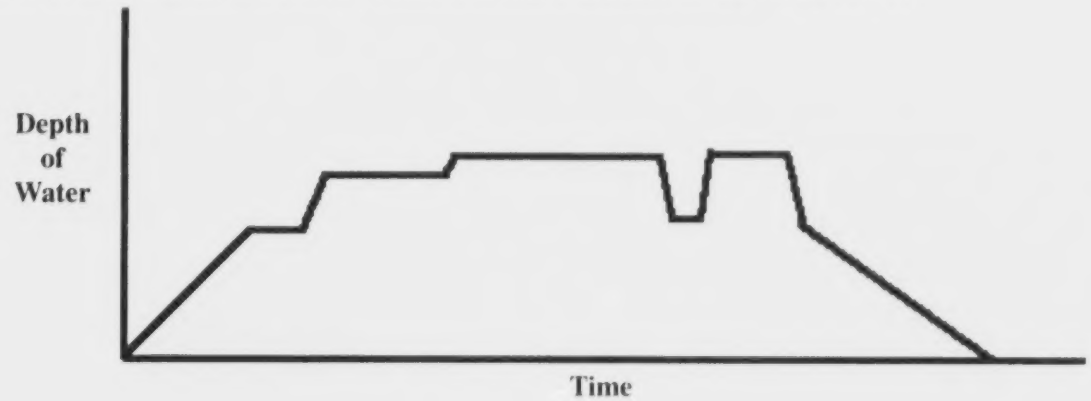
Suggestions for Assessment:

- Students work in pairs to draw a line graph explaining a real-world phenomenon, then present and explain the completed graph to the class. A partial listing of possibilities could be given, such as number of customers at a department store, points scored in a football game, time spent on homework throughout the year, attendance at sporting events, yearly chocolate sales, annual flower sales. Compare data for each topic to time of day, time of month, or time of year, depending on which is most relevant.

Activity Three Handout

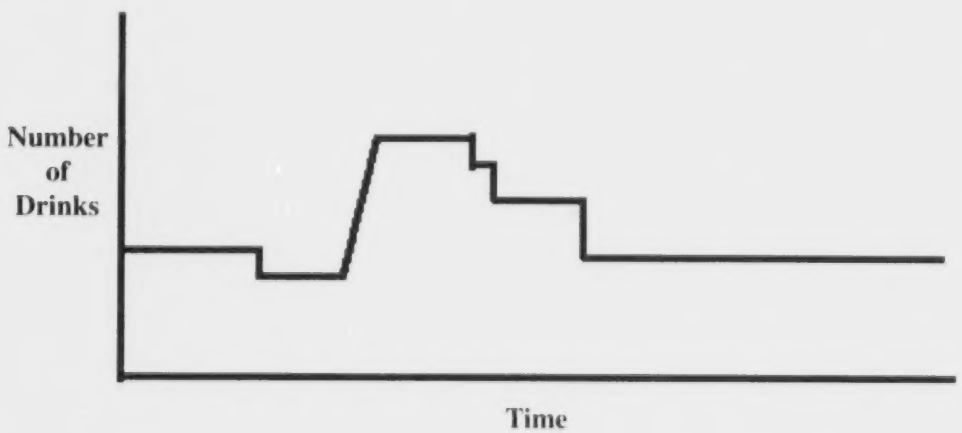
A Bathtub Problem:

- Use your imagination to create a possible scenario depicted by the graph below:



A Drink Machine Problem:

- Use the graph above to create a story describing the situation at the beverage dispensing machine below.



Activity Four: Match It Graph It

Teacher Notes

Real World Math With the CBL System by Chris Brueningsen, et al., Texas Instruments, Inc., 1994, pp. 101-105.

- This is a wonderful Texas Instruments CBL activity. Students will transfer the program into the graphing calculator using Graph Link. The activity itself may be used at multiple levels by dividing it into sections. Basically it gives a graph and asks students to emulate the graph using the motion detector.

Activity Five: Creating a Pie Chart on the TI-82

Teacher Notes

- The program PIECHART for the TI-82 can be entered into the graphing calculator from the following resource: *TI-82 Graphing Calculator Activities for Middle School Math.* Lund, Charles. MathWare, P.O. Box 3025, Urbana, IL 61801

- Since pie charts provide an immediate sense of equal or unequal distribution, students often like the result, if not the means needed to achieve it. Using the PIECHART program, teachers can simplify the process and keep interest in the information to be gained from this format.

Sample Problem:

- Ask students which months they were born. Tally the results by months. Transfer data to list one by pressing **<STAT>** **<1>** (EDIT) and then typing data into the first list. Then use the PRGM key to select the PIECHART program. Select the data display that corresponds to your information, since the program allows for percentages or raw data.

Adaptations:

- Students create a budget for a family for one month including, but not limited to, food, clothing, mortgage, rent, car, miscellaneous, utilities, insurance, phone. The monthly car and rent payments can be taken from a newspaper. Some might prefer to actually call car dealerships or real estate brokers. Looking at the pie graph, students can answer such questions as "Which category consumes most of the budget?" Students could also transfer the pie chart to a coordinate axis. Using each quadrant as 90 degrees, estimate percentages for each part of your budget. Given various annual incomes, students could also estimate a monthly budget.

Suggestions for Staff Development:

- Welcome and brief overview of workshop (5-10 minutes)
- Engage participants in the shoe size activity and give instructions for entering information into lists and forming a scatter plot in the TI-82 (20-30 minutes)
- Activities (Teachers will be divided into three groups) (30-45 minutes)
  - √ All groups will be given copies of Activities One, Three, and Four
  - √ Groups will be given the necessary materials and equipment and instructed to return in 45 minutes for discussion.
- Answer the following questions in a discussion format. (15-20 minutes)
  1. How did the knowledge that you needed to present influence your thinking?
  2. How do you think students would relate to these activities?
  3. What are the benefits of the different activities?
  4. What is the value of having students use paper and pencil as well as the calculator in considering graphs?
- Break (10-15 minutes)
- Groups adapt the scatter plot activity to show how activities can be used at higher levels. (15-20 minutes)
- Total Group Discussion of Assessment (10-15 minutes)
  - √ How do you assess a student's work?
  - √ What are some characteristics of "good" problems?
  - √ How would you have students demonstrate interpretive skills using technology?
  - √ How do you create a problem that piques the interest of students?
- If time permits, complete Activity Five with the total group. (15 minutes)
- Closure (5-10 minutes)
  Sample discussion questions:
  - √ Which of the Standards are we addressing with problems of graph interpretation?
  - √ What training do you need to pursue this technological focus?

Lesson 9: Speeding—A Study in Linear Functions

Authors

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Approximate Time Needed For Lesson

One to five traditional class periods, depending on how many activities in the lesson are completed

Content Area

Patterns, Functions, and Algebraic Thinking

Process Standards

Problem Solving, Connections

Goals for Students

- To perform a linear regression to describe a functional relationship
- To apply skills involving linear functions to analyze and describe real-world situations

Description or Summary of Lesson

Students in Math Tech II, Algebra I, Algebra II, and Advanced Algebra are the intended audience. It is assumed that students can calculate slope, graph a linear function, and write an equation for a linear function. Prerequisite skills will be reviewed in a handout, and this activity also will teach students how to perform the graphing calculator skills needed to complete this activity. Through the use of several activities, students will learn and apply basic knowledge of linear functions to a problem involving speeding tickets.

Background Information

This lesson involves applying earlier concepts related to linear functions to a real-world situation that is of high interest to secondary students. Some or all of the activities may be used, depending on the math background of the students. Extensions are included, which make it also appropriate for more advanced students.

Equipment and Materials

- View screen with graphing calculator
- Overhead projector
- Each student will need:
  - √ access to a TI-82 or TI-83 graphing calculator
  - √ several sheets of graph paper and a straight edge
  - √ handouts

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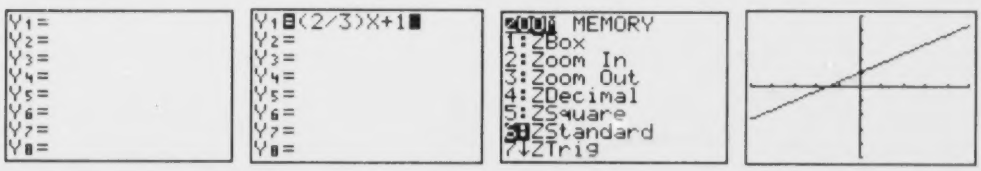
Activity One: Prerequisite Review

Teacher Notes

TI-82 Instructions

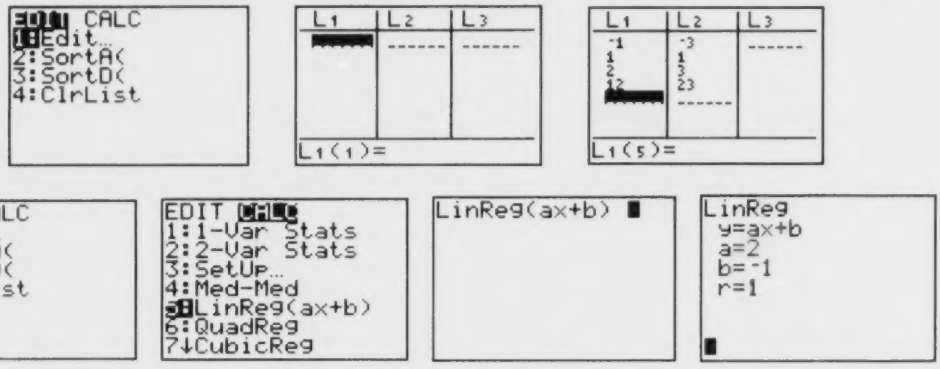
Skill One: Graphing Lines (see problem 6 on the Activity One Handout).

- Press the <**Y=**> key.
- Type the equation. Use the < **X,T,θ** > key for any variables used. Be sure to place parentheses around any fractions.
- Push the <**ZOOM**> key.
- Press <**6**> (ZStandard).
- If the graph on your calculator is not acceptable or does not show up, press the <**WINDOW**> key and change the Xmin, Xmax, Ymin, and Ymax settings.



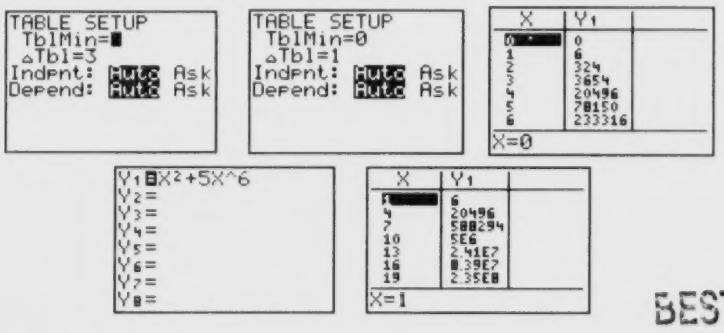
Skill Two: Regression (see problem #9 on the Activity One Handout).

- Press the <**STAT** > key.
- Press <**1**> (Edit).
- If there are numbers in the table already, you will need to erase them. Use the <▲> key and highlight the L_1. Press the <**CLEAR**> key. Then, press the <**ENTER**> key. This will erase everything in this column. If there is data in the other column, press the < ▶> key to get to the next column and repeat the procedure.
- Go to the L_1 column using the ▶, ◀, ▲, or ▼ arrow keys. Type the first x-coordinate. Then, press <**ENTER**>. Type the 2nd x-coordinate and press <**ENTER**>. Repeat the procedure.
- When you have finished typing your x-coordinates in the L_1 column, press the < ▶> key to go to the L_2 column. Type the 1st y-coordinate and press <**ENTER**> key. Type the 2nd y-coordinate and press <**ENTER**> key. Repeat the procedure.
- (***Note:*** Be sure that the L_1 and L_2 columns are the same length.)
- When you are finished typing your y-coordinates, press the <**2nd**> < **MODE**> (QUIT) keys.
- Press the <**STAT**> key.
- Press the <▶> key to go to the CALC menu.
- Press <**5**> for "LinReg(ax+b)" to find the linear regression equation. LinReg(ax+b) will appear on the calculator screen. Press <**ENTER**>.
- The answer will now appear on the screen.
- (***Note:*** If you get an error message that says "ERR: DIM MISMATCH," your L_1 and L_2 columns are not the same length.)



Skill 3: Make a chart (see problem #10 on the Activity One Handout).

- Press the <**Y=**> key.
- Type the equation that you wish to use.
- Press <**2nd**> <**GRAPH**> (TABLE).
- If your table does not start with the number you desire OR the *x*-value of the chart does not increase in the increments you want, press the <**2nd**> key. Then, press the <**WINDOW**> key (TblSet).
- You will see a TABLE SETUP menu.TblMin is the number where your table will start. Type the number where you want the table to begin and press <**ENTER**>. ΔTbl is the amount of change from one *x*-value to the next. (In other words, it is how much the values in the *x* column go up or down.) Type the number that you want for ΔTbl and press <**ENTER**>.
- Press <**2nd**> <**GRAPH**> to see your new table.
- Press the <▲> key or the <▼> key to see more entries in the table.



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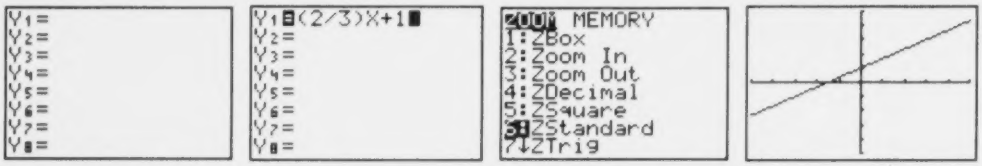
TI-83 Instructions

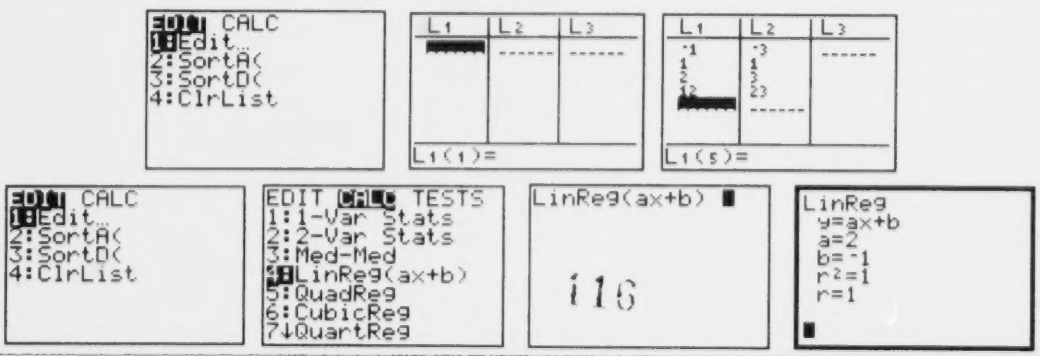
Skill One: Graphing lines (See Problem # 6 on the Prerequisite worksheet).

- Press the <**Y=**> key.
- Type the equation in Y_1=. Use the <**X,T,θ,n**> key for any variables used. Be sure to place parentheses around any fractions.
- Push the <**ZOOM**> key.
- Press <**6**> for ZStandard.
- If the graph on your calculator is not acceptable or does not show up, press the <**WINDOW**> key and change the Xmin, Xmax, Ymin, and Ymax settings.



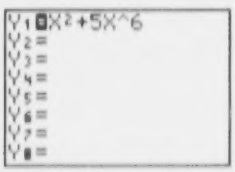
Skill Two: Regression (see problem # 9 on the Activity One Handout).

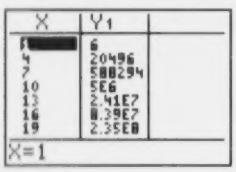
- Press <**STAT**>.
- Press <**1**> for Edit.
- If the names for the columns are L_1, L_2, and L_3 go to Step 4.
- Otherwise, press the <**STAT**> key. Press <**5**> for SetUpEditor. Then, press <**ENTER**>. Press the <**STAT**> key and then press <**ENTER**>.
- If there are numbers in the table already, you will need to erase them. Use the <▲> key and highlight the L_1. Press the <**CLEAR**> key. Then, press <**ENTER**>. This will erase everything in this column. If there is data in the other column, press the <▶> key to get to the next column and repeat the procedure.
- Go to the L_1 column using the arrow keys. Type the first x-coordinate. Then, press <**ENTER**>. Type the 2nd x-coordinate and press <**ENTER**>. Repeat the procedure.
- When you have finished typing your x-coordinates in the L_1 column, press the <▶> key to go to the L_2 column. Type the 1st y-coordinate and press <**ENTER**>. Type the 2nd y-coordinate and press <**ENTER**>. Repeat the procedure.
- ***Note:*** Be sure that the L_1 and L_2 columns are the same length.
- When you are finished typing your y-coordinates, press <**2nd**> <**MODE**> (QUIT)
- Press <**STAT**>.
- Press the <▶> key to go to the CALC menu.
- Press <**4**> for "LinReg(ax+b)" to find the linear regression. LinReg(ax+b) will appear on the calculator screen. Press <**ENTER**>.
- The answer will now appear on the screen.
- ***Note:*** If you get an error message that says "ERR: DIM MISMATCH", your L_1 and L_2 columns are not the same length.
- Sometimes, the r and r^2 value will not appear on the screen. If you want to see these correlation coefficients, press the <**2nd**> key and then press the <**0**> (CATALOG). Press the <▶> key until you get to DiagnosticOn.
- Press <**ENTER**> when this is highlighted. Press <**ENTER**> again. Now press <**STAT**>, <▼> to CALC, and <**4**> for LinReg(ax + b). The r and r^2 values should now appear.

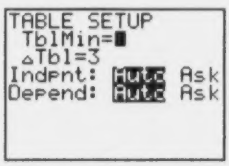


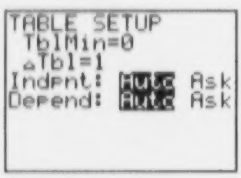
Skill 3: Make a Chart (see problem # 10 on the Activity One Handout).

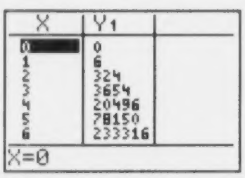
- Press the <**Y=**> key.
- Type the equation you wish to use.
- Press <**2nd**> <**GRAPH**>(TABLE).
- If your table does not start with the number you desire OR the *x*-value of the chart does not increase in the increments you want, press the <**2nd**> key. Then, press <**WINDOW**> (TblSet)
- You will see a TABLE SETUP menu. TblMin is the number where your table will start. Type the number where you want the table to begin and press <**ENTER**>. ΔTbl is the amount of change from one *x*-value to the next. (In other words, it is how much the values in the *x*-column go up or down.) Type the number you want for ΔTbl and press <**ENTER**>.
- 6. Press <**2nd**> <**GRAPH**> (Table) to see your new table.
- 7. Press <▲> or <▼> to see more entries in the table.











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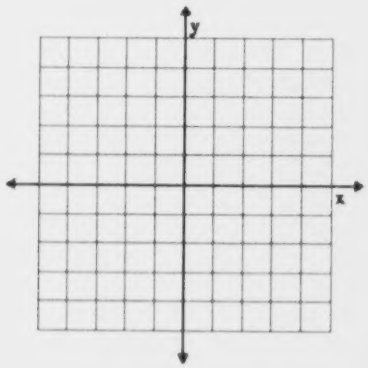
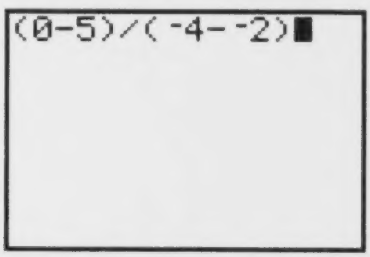
Activity One Handout: Prerequisite Review

1. Plot the following points: A(2,5), B(-1,3), C(0,2), and D(4,-3).



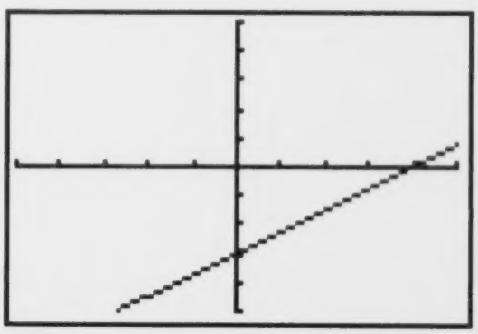
2. Calculate the slope of the line through the points (4,-5) and (1,3) algebraically.

3. Using your graphing calculator, compute the slope of the line through the points (-2,5) and (-4,0).

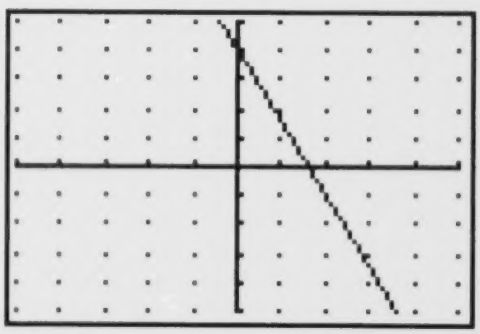


4. Calculate the slopes of these two lines using $m = \frac{\text{rise}}{\text{run}}$.

a.

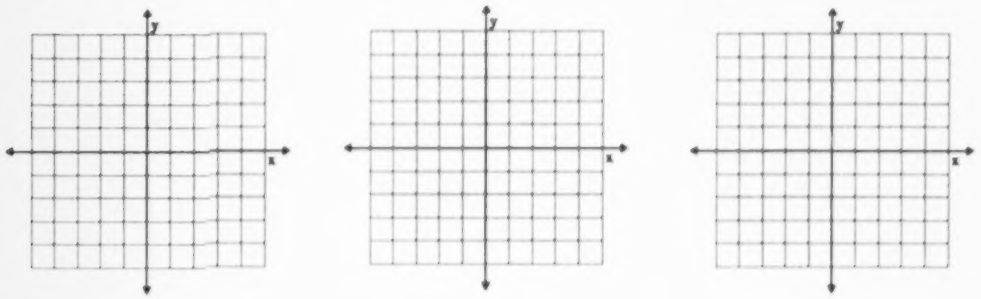


b.



5. Graph the following lines: a. $y = 2x + 3$, b. $y = \frac{2}{3}x + 1$, and c. $2x + 3y = 6$.

a. b. c.



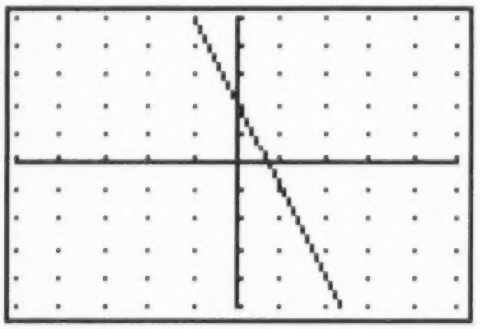
6. Using a graphing calculator, sketch the following lines:

a. $y = 3x - 4$ and b. $y = \frac{4}{3}x + 2$.

a. b.

7. Write the equation of the line through the points (1,5) and (9, 2).

8. Find the equation of the line in the following graph using the slope and the *y*-intercept of the line.



9. Using your graphing calculator, find the equation of the line through the following points using linear regression: (-1,-3), (1,1), (2,3), and (12,23).

10. Using your graphing calculator, complete the table for $f(x) = x^2 + 5x^6$.

x	y
0	
1	
2	
3	
4	
5	
6	
7	
8	

Activity Two Handout: An Introduction to Speeding Tickets

You are traveling on I-40 in Tennessee and you are stopped by a state trooper. To your horror, you are given a speeding ticket. In Tennessee, the cost of your ticket is determined by the city and/or county in which the ticket is issued.

- Suppose you are stopped for speeding in the Tennessee city of Petersburg. The Petersburg City Council has decided the cost of a speeding ticket on the interstate (with a speed limit of 65) will be determined using the following function:

$$T = 80 + 1(S - 65)$$

S is the speed you are traveling in miles per hour (mph) and T is the cost of the ticket.

1. What would be the cost of your speeding ticket if you are traveling 72 mph? Explain how you arrived at your answer.

2. Find T if $S = 55$.

3. Explain why you would not get a traffic ticket if you are traveling 55 mph, even though the answer in #2 indicates your speeding ticket is a certain amount.

4. For what values of S would you not use this formula? Why?

5. Your teacher received a $100 speeding ticket in Petersburg. How fast was he or she going? Explain how you arrived at your answer.

6. The city of Petersburg has hired you to establish a way for city officials to quickly determine the cost of a traffic ticket. You determine that a chart would be the solution to this problem. Using your graphing calculator, make a speeding ticket chart for speeds between 66 mph and 90 mph.

Speed (mph)	Cost of Ticket
66	
67	
68	
69	
70	
71	
72	
73	
74	
75	
76	
77	
78	
79	
80	
81	
82	
83	
84	
85	
86	
87	
88	
89	
90	

7. Look at the chart above and the function $T = 80 + 1(S - 65)$. Why do you think the number 1 is included in the function?

8. All other streets in Petersburg have a speed limit of 45 mph. Suppose you have been hired as a consultant by the City Council to establish a formula that determines the cost of a speeding ticket on these streets. They have given you this stipulation:

The minimum ticket should be $30.

- Create a fair and reasonable function using S and T. Explain how you arrived at your function, and explain why it is fair and reasonable.

Activity Three Handout: When is a Ticket the Same?

Edithtown and Keelston, two fictional cities in Tennessee, use the following functions to determine the cost of speeding tickets:

Edithtown: $T = 66 + 2(S - 65)$ **Keelston:** $T = 10 + 10(S - 65)$

1. Using a graphing calculator, create a chart that lists the cost of speeding tickets in Edithtown and Keelston for speeds between 66 and 90. At what speed is the cost of a speeding ticket the same in both cities? Circle this speed on the chart you create.

Edithtown

Speed (mph)	Cost of Ticket
66	
67	
68	
69	
70	
71	
72	
73	
74	
75	
76	
77	
78	
79	
80	
81	
82	
83	
84	
85	
86	
87	
88	
89	
90	

Keelston

Speed (mph)	Cost of Ticket
66	
67	
68	
69	
70	
71	
72	
73	
74	
75	
76	
77	
78	
79	
80	
81	
82	
83	
84	
85	
86	
87	
88	
89	
90	

2. For every additional mph you go over the speed limit in Edithtown, what happens to the cost of the speeding ticket? (Hint: use the chart in #1)

3. Write $T = 66 + 2(S - 65)$ in slope-intercept form ($y = mx + b$). What is the slope of the equation? What does the slope mean? What are its units?

4. For every additional mph you go over the speed limit in Edithtown, what happens to the cost of the speeding ticket?

5. Write $T = 10 + 10(S - 65)$ in slope-intercept form. What is the slope of the equation? What does the slope mean? What are its units?

6. Explain how slope and the change in the cost of the speeding ticket are related.

7. Using a graphing calculator, sketch the speeding ticket functions. Describe the window that you used.

   Edithtown Keelston Both cities on the same axis

8. Describe how the graphs are similar. Why do you think they are similar?

9. Describe how the graphs are different. Why do you think they are different?

10. An interesting situation occurs when you look at the speeding ticket functions for these two cities. There is only one speed that you could travel in either city and get a ticket that costs the same amount in both cities. There are at least three ways to attack this problem.

    a. Algebraic Solution: Solve the system of equations for the two cities of Edithtown and Keelston. Show all work.

    b. Graphical Solution: Graph the two functions on your graphing calculator. When do they intersect? Justify your answer (i.e., show the graphs, the point of intersection, and the window).

    c. Numerical method: You did this already in problem #1.

    d. Can you think of another way to solve this problem?

Activity Four: Piecewise and Other Functions

Note: The data used in this section were obtained from police departments in July 1997.

Though some real towns and cities in Tennessee actually use linear functions to determine the cost of speeding tickets, this is not always the case.

- For example, the city of Martin uses the following schedule:

Traffic Offense	Cost
Speeding (Simple)	$56.00
Speeding (Radar-Limit + 15 mph)	$56.50
Speeding (Radar-Limit + 20 mph)	$61.50
Speeding (Radar-Limit + 25 mph)	$66.50
Speeding (Radar-Limit + 30 mph)	$71.50

- This is a stepwise function.

1. Make a graph that represents the speeding fines in Martin, Tennessee.

2. What would be the cost of a speeding ticket in Martin if your speed in a 55 mph zone is 72 mph?_____ 80 mph?_____ 85 mph?_____ 92 mph?_____ Explain how you arrived at these answers.

3. Develop a piecewise function for this schedule.

- Another example of speeding fines in a 55 mph speed limit zone is given below in the information from Weakley County, Tennessee.

Speed (mph)	Ticket Cost
60–64	71
65–70	120
71–75	125
76–80	130
over 80	155

- This is another piecewise function.

4. Explain the pattern for speeds 65 mph through 80 mph?

5. Why do you think the $71 fine does not fit the pattern?

6. Why does the $155 fine not fit the pattern?

7. Why do you think there is no listing for speeds from 56 to 59 mph?

If a Tennessee state trooper issues a speeding ticket to a motorist, the exact amount of the the ticket is determined by the county in which the ticket is issued. Some of the factors used to determine the value of a speeding ticket include the following: state litigation tax, county clerk fee, local county tax, law library fee, and an officer or trooper fee.

If you are ticketed by a police officer in Metropolitan Nashville/Davidson County, the cost of a speeding ticket is simply $78.50 for exceeding the speed limit on the interstate. At the discretion of the police officer, a fine for reckless driving may be issued with a maximum value of $500 for traveling 15 mph over the speed limit.

However, if a speeding ticket is issued by a state trooper, the amounts are $69 for speeds of 74 mph or less and $113.50 for speeds of 75 mph or more. These interstate fines are the same whether the speed limit is 55 mph or 65 mph.

8. For what speeds is a ticket more costly if you are stopped by a police officer, and for what speeds is it more costly if you are stopped by a trooper?

9. Do you think this ticketing schedule is fair? Explain your answer.

Some similar functions are used to determine fees for postal rates, package delivery services, car rentals, and cellular phone charges. Can you think of other examples?

10. Further investigate one of the ideas in #9.

Activity Five: Speeding Ticket Projects

Select one of the following projects to complete. You may work with a partner.

1. Contact city, county, or state police departments in your area. Find out how the cost of a speeding ticket is determined in at least 3 different governmental entities. Write a paper on the fairness of these fines.

2. You have been hired by a city that wants to deter people from speeding. You have been asked to create a function that is not linear and would deter motorists from speeding. Create a function and explain why your function would accomplish this goal. Use charts and graphs to support your claim.

3. Conduct and report research on the correlation between speeding and vehicular accidents.

4. Conduct and report research on speed limits in neighboring states.

5. Contact city, county, or state police departments in your area. Determine their definition of "reckless driving." How is this determined and what is this fine?

Assessment and Inservice Suggestions

Assessment:

- Handouts provided in this lesson can be graded by the teacher.
- Teacher can observe learning while students are working.
- Students could be asked to write a paper describing what they have learned in this lesson.
- Students could complete a project as suggested in the extensions.

Inservice:

- Teachers can get graphing calculator experience by completing the handouts in this activity as part of an inservice program.
- The TI-82 and TI-83 instructional sheets will be extremely valuable for teacher with little or no graphing calculator experience.
- Teachers should get the "hands-on" experience of calling police departments to research the cost of speeding tickets in their area.
- Teachers should get an opportunity to brainstorm to consider other ways to enhance this activity. For example, one discussion among teachers produced an idea to invite a police officer to speak in a class.

Lesson 10: Traveling Through the Solar System— An Exploration of Mass vs. Weight

Authors

★ Jane Hampshire, St. Agnes Academy, Memphis, TN
★ Elizabeth Greiner, Frayser HS, Memphis, TN

Approximate Time Needed for Lesson

One to two traditional class periods or one 90-minute period

Content Area

Statistics and Probability

Process Standards

Problem Solving, Communication, and Connections

Goals for Students

- To use calculator and non-calculator methods for analyzing data
- To apply data collection and analysis in a study of mass and weight
- To use the concept of line of best fit to calculate weight (mass) when given mass (weight)

Description or Summary of Lesson

Students in Arithmetic, Pre-Algebra, General Science, and Physical Science are the intended audience. Using appropriate technology and working in small groups, the students collect data by measuring the mass of various objects, calculating the weights, and representing the data in various ways. Using calculator and non-calculator methods, the students will analyze the data to determine relationships, make predictions using a line of best fit, and formulate written conclusions.

Background Information

This is an introductory lesson intended for students in grades 7-9. Students are assumed to have mastered graphing in a plane, be familiar with slope and *y*-intercept, and be familiar with basic graphing calculator keys. There are references for extending and connecting to Algebra 1, Algebra 2, Pre-Calculus, and Calculus.

Materials and Equipment

- Balance scale
- A few uncooked spaghetti noodles
- Centicubes or other gram weights
- Glue
- Raisins
- Graph paper
- Construction paper
- Overhead projector and calculator, with view screen if possible
- Each student will need:
  - √ notebook paper and pencil
  - √ access to a TI-81, TI-82, or TI-83 graphing calculator
  - √ handouts

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Activity One: Raisins—Collecting Data to Explore Mass

Teacher Notes

Outline:

- Students measure the mass of different amounts of raisins.
- Students use Activity One Handout to record and graph the results by hand and with the calculator.
- Students analyze the data and complete the handout.
- Students are given a homework assignment to name five other situations that make a linear relationship besides the number of items vs. mass.
- Students complete the following evaluation problem:
  Use your calculator to make a scatter plot and line of regression for the following data:

number of paper clips	weight
20	22
30	30
40	46
50	54
60	67

Show your graph as your final answer.

Extensions:

- A more advanced class could compute the line of regression and determine the correlation coefficient.
- You can repeat the above using Tootsie Rolls, paper clips, or similar objects.
- The following books will help you use a CBL to measure and form the lists for such an experiment. The specific activities listed all use a force sensor. The first reference is very similar to the raisin activity; the others are designed for more advanced places in the curriculum.

Activity #2 "Making Cents of Math" in *Real Word Math with the CBL System* by Chris Brueninger, Bill Bower, Linda Antinone, Elisa Bruening, copyright 1994 by Texas Instruments.

Activity #6 "Exploring Linear Equations II" in *CBL Explorations in Algebra for the TI-82 and TI-83*, copyright 1996 by The Meridian Creative Group

Activities #12 "Modeling Geometric Sequences II" and #17 "Modeling Vector Addition" in *CBL Explorations in Pre-Calculus for the TI-82*, copyright 1995 by The Meridian Creative Group

Activities #11 "Vector Analysis of Forces" and #14 "Moments about a Fixed Point" in *CBL Explorations in Calculus for the TI-82*, copyright 1995 by The Meridian Creative Group

Activity One Handout: Raisins

1. Find the measures of the following using a balance scale. Place the given number of raisins on one side of the balance. Place centicubes (or other gram weights) on the other side of the balance until both sides are the same. The raisins weigh the number of centicubes. Remember 1 centicube has a mass of 1 gram. Write the results in the table.

Number of raisins	Mass in grams
10	
20	
25	
30	
40	

2. What is the independent variable? _____

3. What is the dependent variable? _____

4. Are these variables related? How? _____

_____

5. On graph paper, graph the number of raisins vs. the corresponding mass. Choose an appropriate scale.

6. Do the data points show any relationship? What is that relationship?

_____

_____

7. Is there a positive correlation (that is, does the dependent variable increase/decrease when the independent variable increases/decreases) or negative correlation (that is, does the dependent variable increase/decrease when the independent variable decreases/increases) or no correlation (no consistent relationship)? _____

8. Even though the data points may not be in exactly one straight line, there may be a line that is close to most, if not all, points. This is called the line of best fit. Use the spaghetti to find this line of best fit. Does the spaghetti intersect with every point, come close to most points, or not intersect any point?

_____

_____

9. What does the slope represent in this problem? _____

_____

10. What number value is the slope? To find the slope, choose any two of the data points that fall on or very close to the spaghetti and use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

    Slope = _____

11. According to the data, what should be the *y*-intercept? _____

12. What is the *y*-intercept according to the line of best fit (spaghetti)? _____

13. Is the value of #11 different from the value of #12? Why/Why not? _____

    _____

14. Write an equation for the line made by the spaghetti in the form $y = mx + b$ where m is the slope and b is the *y*-intercept. _____

15. Predict the missing values in the table by using the linear equation you found in #14.

Number of raisins	Mass in grams
1000	
4	
	16
	95

16. Make a real-life graph. On a piece of heavier paper (construction paper, poster board), create an appropriate title for this graph. Draw axes for the first quadrant. Draw the hash marks along the axes and choose an appropriate scale. Glue the raisin pieces at the data points from Question 1. Glue on spaghetti as the line of best fit. Label the line of best fit with the equation from Question 14.

17. Turn on the graphing calculator.

    - Press <**2nd**> < **Y=**> (Stat Plot).
    - Press <**4**> (Plotsoff). This returns the cursor to the home screen.
    - Press <**ENTER**>. This will turn all of the stat plots off.
    - Press <**Y=**>. If any of the = signs are highlighted, move the cursor on top of the = and press <**ENTER**>. Doing this will make the graph of this equation invisible. OR if you do not want that equation anymore, keep the cursor on top of the equation and press <**CLEAR**>. Do one of these two things for any equation that is in any Y.

- Press <**STAT**>. Press <**ENTER**> for the Edit menu. In the first list or column of the table (L_1 on the TI-82 or X on the TI-83) enter the data for the number of raisins that are weighed. After each entry, press <**ENTER**>. Arrow to the right to the second list, L_2. If it is not named, name it L_2 by typing <**2nd**> <**2**> (L_2). Enter the data that was collected for mass in grams for each amount of raisins.

- Press <**2nd**> <**Y=**> (Stat Plot) and then <**ENTER**> with the cursor on Plot 1. Press <**ENTER**> while the cursor is flashing on top of the ON.

- Press <▼> once. Move the cursor on top of the scatter plot. Press <**ENTER**> to select this type of graph. Arrow down once to Xlist.

- TI-82: Press <**2nd**> <**1**> (L_1). Press <▼> and select L_2 for Ylist.

  TI-83: Enter X for Xlist. Press <▼> and <**2nd**> <**2**> (L_2) for Ylist.

- Press <▼> and choose the type of mark you want to represent your data.

- Press the <**WINDOW**> key. The window sets up the view screen where the graph will appear. Be sure to use numbers that include all data points. Buffer these points on all sides in order to see a complete graph . Xmin is the smallest value for the viewscreen. Use 0 for the Xmin in order to see the *y*-axis.

- Press the <▼>. The Xmax is the largest number of raisins. Since the largest number of raisins on the table is 40, buffer this by 20 and type in 60.

- Press the <▼>. The Xscl (*x*-axis scale) sets the tic marks on the axis. Press < **5**>.

- Press the <▼>. The Ymin is the smallest value of the masses. In order to see the *x*-axis, type in 0. <▼>. The Ymax is the highest value of the masses. Using a buffer again, type in 20 higher than the highest mass value. <▼>. Determine the Yscl (*y*-axis scale) by dividing the largest mass value by how many tic marks you want to see. (Suggestion: type in 5)

- Press <**GRAPH**>. To look at the values of the points, press <**TRACE**>. A flashing cursor will appear on top of the first data point. At the bottom of the screen, X= and Y= will appear. To see the rest of the data points, press the <▶> key.

- To find the line of best fit, press the <**STAT**> key. To calculate this equation, arrow once to the right to CALC. The equation forms a line, so press <**4**> (LinReg (ax+b)), which is a linear regression. LinReg(ax+b) appears on the home screen. Press <**ENTER**>.

18. The calculator finds the value of the slope (a) and y-intercept (b). (Round to the hundredths place)

    Slope=_____

    Y-intercept=_____

    One equation for a line is $y = mx + b$ (m=slope and b=y-intercept)

19. What is the equation of this line? _____
    (Hint: use substitution)

20. To look at this graph, press **<Y=>**. Type in the equation that you wrote above. The window still contains the data points from above. Press **<GRAPH>**.

21. How do the calculator slope and y-intercept compare with the slope and y-intercept from the spaghetti and pencil and paper calculations?

    _____

    _____

22. Name 5 new concepts that you learned from the experiment and/or 5 things that you now understand better because of the experiment.

    _____

    _____

    _____

    _____

    _____

23. Write a journal entry that includes the answers to the following questions.
    a. Summarize the experiment.
    b. What relationship did the centicubes have with the mass?
    c. What conclusions can be made about the number of raisins and their mass?
    d. What did the line of best fit (spaghetti) tell about this relationship?
    e. How would someone find the mass of 100 raisins?
    f. How would someone find out how many raisins are in 200 grams?

Activity Two: Weight Through the Solar System

Teacher Notes

Outline:

- Teacher and students agree on formula: weight = mass **x** gravity.
- The teacher will choose a celebrity and guide all students through an example using the steps on the student handout.
- Using the handout, small groups perform same calculations for a second celebrity.
- Homework assignment: Have students find their own weight on all planets and graph resulting data.

Extension:

Given that the moon's gravitational pull is 1/6 that of the earth, have students find the celebrity's weight on the moon. Is this comparable to any planet? Who would use this information?

Hopefully, your students will identify with, or at least readily recognize, the people you send through the solar system in the last activity. You may wish to choose important people in your school or community. We have included many athletes since their weights are readily available on the Internet (just search for a name you wish to include). Following is a sample list of people whose weights were found on the internet:

Lisa Leslie	165 pounds
Rebecca Lobo	185 pounds
Brett Favre	225 pounds
John Stockton	175 pounds
Jackie Joyner-Kersee	154 pounds
Oprah Winfrey	150 pounds
Anfernee Penny Hardaway	210 pounds

Suggestions for assessment:

- Journal entries
- Graphs
- Tables
- Participation in class discussion
- Oral group summaries
- Homework assignment
- A quiz

Activity Two Handout

1. Choose a celebrity from the list to discover his/her weights on other planets.

2. On which planet would you expect the celebrity you choose to weigh the most?

   _____

3. On which planet would you expect the celebrity you choose to weigh the least?

   _____

4. Calculate the values for the following table. The following formula will be useful:

Planets	Mass of Person	Gravity	Mass*Gravity	Weight
Mercury		0.38		
Venus		0.91		
Earth		1		
Mars		0.38		
Jupiter		2.54		
Saturn		0.93		
Uranus		0.8		
Neptune		1.2		
Pluto		0.06		

   (Assume that gravity values are in terms of Earth = 1)

5. a. Create a bar graph using the results of the data showing the weight of the person on each planet.
   b. Give the bar graph a title. Label the *x*-axis as the planets and the *y*-axis as the weight of the person. Choose an appropriate scale.

6. Turn on the graphing calculator. Make a bar graph on the calculator using the same data. Use the following steps to create this bar graph. All bold type words are keys on the calculator.

   - Press the <**STAT**> key. With EDIT highlighted, and press <**1**> (Edit) and <**ENTER**>.
   - In the first column or list in the table (L_1 on the TI-82 and X on the TI-83) list the numbers from 1–9. After each number that is typed into the L_1 or X list, press <**ENTER**>. These numbers represent the planets, where 1 is Mercury and 9 is Pluto.
   - Press the right arrow key once and the cursor should now be under L_2 .List the weights of the person in the chart above in the same order as the nine planets.
   - Press <**Y=**> and clear any Y= that is not clear. Press <**2nd**> <**Y=**> (STAT PLOT). In the STAT PLOT menu, with PLOT 1 highlighted, press <**ENTER**>. Turn the stat plot on by pressing <**ENTER**> while the cursor is flashing on top of ON. Press <▼> once. Highlight the bar graph and press <**ENTER**>.
   - Press the <▼> once. The Xlist is the horizontal axis, which will be the planets. Make sure that List 1(L_1 on the TI-82 or X on the TI-83) is placed in this spot. If L_1 or X is not present, place the cursor on top of what is there and press <**2nd**> <**1**> (L_1) or on the TI-83, press <**x,t, ,n**>.
   - Press the <▼> once. The Freq (frequency) is the vertical axis, which will represent the weight of the person on the given planet. Make sure that L_2 (List 2) is placed in that spot. If L_2 is not present, place the cursor on top of what is there and press <**2nd**> <**2**> (L_2).
   - Press <**WINDOW**> to set the viewscreen where the graph will appear. Be sure to use numbers that include all of the data points. Buffer these points on all sides in order to see the complete graph. Xmin is the smallest value for the viewscreen. Use 0 for the Xmin in order to see the y-axis.
   - Press the <▼> once. The Xmax is the highest value for the planet number. Since there are nine planets, and in order to see complete bars, type in 10.
   - Press the <▼>. Xscl (x-axis scale) sets the tic marks on the axis. Type in <**1**> because the planets are labeled by the whole numbers 1-9.

- Press the <▼>. The Y-min is the smallest value of the weights. In order to see the *x*-axis, type in 0. Arrow down once. The Ymax is the highest value of weights. In order to see complete bars, add on 20-30 to the highest weight. Arrow down once. The Ysc (y-axis scale) sets the tic marks on the *y*-axis. Determine the scale number by the number used for the scale of the bar graph done by hand. The scale could also be determined by dividing the range of the *y* values by the number of tic marks to be shown.

- Press <**GRAPH**>. The bar graph will appear.

- To look at specific values of the bars, press <**TRACE**>. A flashing cursor appears at the top of the first bar, which is Mercury. At the bottom right of the screen, there is an n=. This is the weight value for the planet. Press the right arrow key. The flashing cursor will jump to the top of the other bars. Look at the n= for the weight value.

7. Which planet produced the highest weight? _____

8. Which planet produced the lowest weight? _____

9. What was the range of the weights? _____

10. What is the relationship between mass and weight? _____

_____

_____

11. What surprised you the most about the differences in the weights? _____

_____

12. Name the planets that have similar gravitational pull. List the reason(s).

_____

_____

_____

13. Name 3 differences between the hand-made bar graph and the calculator bar graph.

_____

_____

14. Name 3 similarities in making a hand-made bar graph and the calculator bar graph.

_____

_____

_____

15. Why would you want to know what you weigh on different planets? _____

_____

_____

_____

16. On graph paper, make a scatter plot using the same data where the independent variable is the planets and the dependent variable is the weight. Give the scatter plot a title and label the axes.

17. Turn on the calculator. Press <**2nd**> <**y=**> (STAT PLOT). Press <**4**> (Plotsoff). This returns the cursor to the home screen. Press <**ENTER**>. This will turn all of the stat plots off. Press <**Y=**>. If any of the = signs are highlighted, move the cursor on top of the = and press <**ENTER**>. Doing this will make the graph of that equation invisible. OR If you do not want that equation anymore, keep the cursor on top of the equation and press <**CLEAR**>. Do one of these two things for any equation that is in any Y=.

- Press <**2nd**> <**Y=**> (STAT PLOT). Press <**ENTER**>. Press <**ENTER**> while the flashing cursor is on top of the ON. Arrow down, highlight the scatter plot , and press <**ENTER**>.

- Arrow down to Xlist and type in L_1 (<**2nd**> <**1**>). Remember L_1 is the list with the number of planets 1-9. Arrow down to Ylist and type in L_2 (<**2nd**> <**2**>). Remember L_2 is the list with your celebrity's weight on the different planets. Arrow down and press <**ENTER**> on top of the type of mark you want to represent the data.

- Earlier, the window was set for the data. Since the data has not changed, leave the window the same.

- Press <**GRAPH**>. The scatter plot will appear. To look at specific point values, press <**TRACE**>. At the bottom of the screen, X= and Y= appear. Use the right arrow to move to points on the right and the left arrow to go back toward the first point.

18. What connection do you see between the bar graph and the scatter plot?

_____

_____

19. Which graph better represents the data? Why?

_____

_____

_____

20. Are there outliers? If so, identify them.

_____

_____

21. Does there seem to be a relationship between a person's weight on a planet and the distance the planets are from the sun? What seems to occur?

_____

_____

22. If you know that, on Saturn, Joe weighs 200 lbs, what does he weigh on Mercury?

Joe's weight on Mercury= _____

23. Write a journal entry that answers the following:
    a. Summarize the experiment.
    b. What is the relationship between mass and weight?
    c. How does gravitational pull affect weight?
    d. Which type of graph better represents the data? Why?
    e. When are the mass and weight the same (if they ever are)?
    f. When you step on the scale, what are you measuring?
    g. If you could choose any planet to live on, which planet would you choose now that you know your weight will change? Why?

Lesson 11: Alien Attack—An Investigation of Quadratic and Higher Equations

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Approximate Time Needed for Lesson

One to two traditional class periods or one 90-minute period

Content Area

Patterns, Functions, and Algebraic Thinking

Process Standards

Problem Solving and Connections

Goals for Students

- To explore a variety of calculator methods for solving quadratic and higher degree equations
- To apply solving quadratic equations in a problem-solving situation

Description or Summary of Lesson

Students in Algebra I, Algebra II, and Pre-Calculus are the intended audience. After a teacher-led discussion of a futuristic problem and graphing calculator methods of solution, the students will apply the methods to solve variations of the problem. They will work in pairs or in groups of three. Also, the same methods will be applied to higher degree equations.

Background Information

This lesson is introductory material with several suggestions for using advanced topics, enrichment, and discussion. Prior to the lesson, students should be able to identify a quadratic, cubic, or quartic equation.

Materials and Equipment

- Graphing calculator (TI-81, TI-82, TI-83, TI-85)
- View screen, if available, and an overhead projector
- Each student will need:
  - access to a graphing calculator
  - one sheet of graph paper
  - four copies of Handout 1
  - Handout 2

Note: The screens used in this lesson are from the TI-82. The TI-83 would use the same procedure. The TI-81 and TI-85 do not have a table method, but the problem can be solved graphically on both and on the TI-85 by using solver.

Activity One: Alien Attack

Teacher Notes

Outline:

- The teacher will work through the futuristic problem with students using the Table and Intersect methods on the graphing calculator and one copy of Handout 1.
- Students work the practice problems using the additional copies of Handout 1 and using both methods.
- Students complete Handout 2.

Note: This lesson may be kept very basic or may be expanded to encompass physics and writing. An element of creativity was used to interest the students and relate the problem to current events.

Teacher Notes for Handout 1

Science Connection:

When an object is propelled upwards, gravity pulls it back to Earth. The graph of time vs. height is a parabolic curve even in a world in which the only force working on the object is gravity, and the path of the object is straight up and down. The relationship of the object and gravity can be expressed by the formula:

$s = h_i + v_i t - 0.5gt^2$, where s is the distance above the starting point, h_i is the starting height, v_i is the initial velocity, t is time elapsed, and g is the acceleration of gravity.

The acceleration of gravity is 9.8 meters/sec^2. With the application of this information, alternative methods of solving quadratics will be developed and practiced.

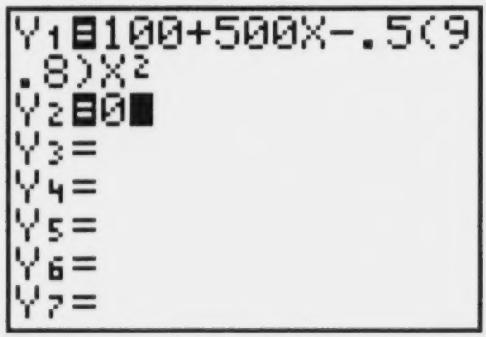
Problem:

An alien ship is hovering above Washington, D.C. at a height of just 100 meters. A giant slime blob is housed in a missile-like container. The alien ship fires the container up into the air at a velocity of 500 meters per second and sets its warp speed to fly away.

1. Let time be represented by x and substitute the given information into the equation. Then change the equation into the form of $y =$. This must occur with any equation to be solved on the graphing calculator.

2. Enter in the equation $y = 100 + 500x - .5(9.8)x^2$.

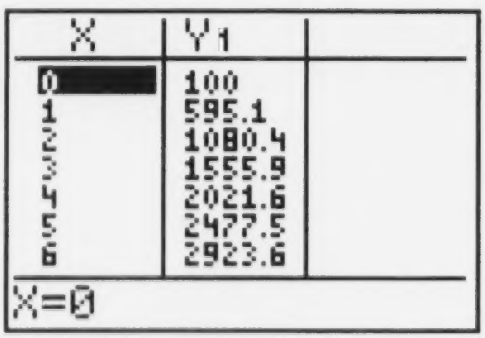
   Press **<Y=>** and enter $Y_1 = 100 + 500x - .5(9.8)x^2$



3. The first approach to be used for solving this problem will be the table feature.

   - Press <**2nd**> <**WINDOW**> (TBLSET)
   - Set TblMin=0
   - Δ Tbl = 1 (This assigns the increments to be used for the *x*-values of the table.)

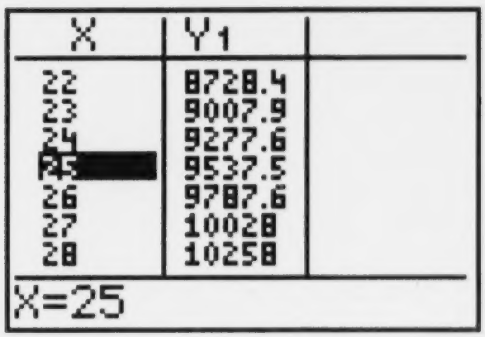
4. Look at the table by pressing <**2nd**> <**GRAPH**> (TABLE).



5. Begin the investigation.

Suggested questions with enrichment and discussion:

1. What is the height of the container at 25 seconds?
   - Use the <▼> to move the cursor along the table's x-values to the desired number, i.e., 25 (short cut: press **<2nd>** **<WINDOW>** and change the Tbl min to 25). Then **<2nd>** **<GRAPH>** (TABLE) to reveal the table.



Now one can see the height (y) is 9537.5 meters.

2. What is happening to the height after 25 seconds?
   It continues to rise.

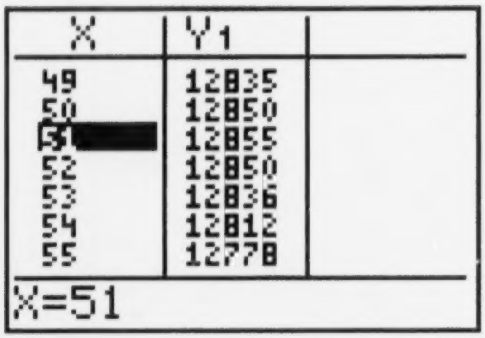
3. What is the *maximum* height the slime blob reaches?
   12,855 meters

Discussion: Highlight that this point is an approximate maximum. It is also the approximate vertex of the parabola. The table value may not be the exact value because the time is in one-second intervals instead of smaller increments.

4. After about how many seconds does the slime blob reach its maximum height?
   51 seconds

Discussion: This point lies on the axis of symmetry.

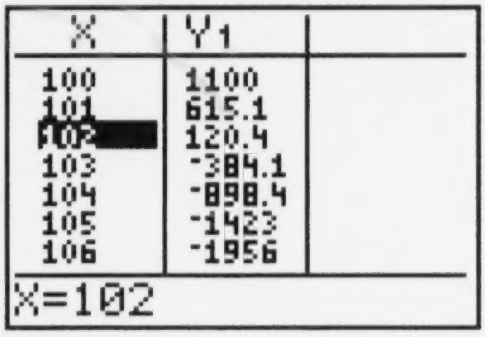
- What is happening on either side of the vertex?
- If a vertical line were drawn through the vertex, what would it show?
- What would the vertical line be called?
- What would the equation of the vertical line be?
- What is happening to the slime blob on each side of the axis of symmetry?
- Write the equation of the parabola formed in the form $y = a(x - h)^2 + k$.
- When the slime blob hits earth what is the ideal y-value?
- When the y-value is 0, the x-value is identified as the zero of the equation. The zero of the equation is actually the solution of the quadratic and higher degree equations.



5. After how many seconds would the blob slime Washington, D.C.?
   approximately 102.24043

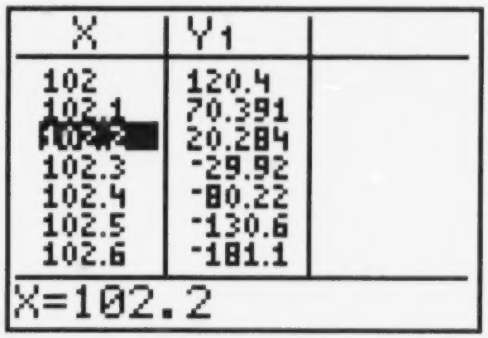
6. What is the height (*y*) when the slime blob is on the ground?
   approximately .08325

7. How close is *.08325* to 0? (Ideally the *y* value should be 0).



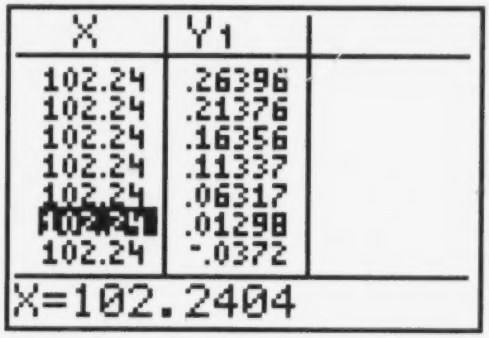
Discussion: Notice in the screen above at 102 seconds the height is positive, and at 103 seconds the height is negative. Therefore the slime blob hits between 102 and 103 seconds after it is hurled into the air.

The **<2nd> <WINDOW>** (TblSet) allows us to change the increments of the table. Change the TblMin to 102 and ΔTbl to 0.1. (See table on the left). You can continue to make adjustments until you find zero for the *y* value. Thus the height is approximately 0 (.08325) after 102.24 seconds. This value was found with TblMin at 102.24 and ΔTbl .0001 (see the following screens).



X	Y1
102	120.4
102.1	70.391
102.2	20.284
102.3	-29.92
102.4	-80.22
102.5	-130.6
102.6	-181.1

X=102.2



X	Y1
102.24	.26396
102.24	.21376
102.24	.16356
102.24	.11337
102.24	.06317
102.24	.01298
102.24	-.0372

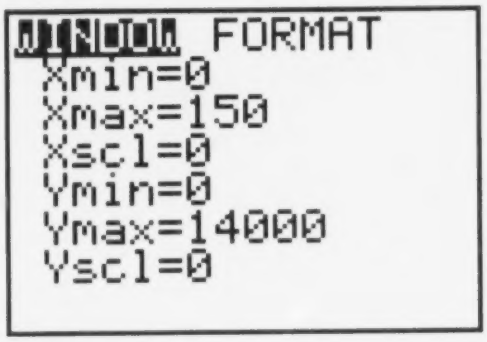
X=102.2404

Enrichment:

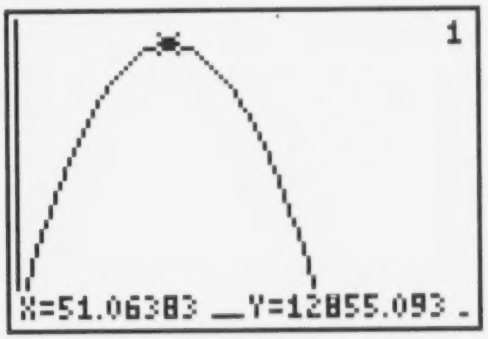
- Make a graph with paper and pencil using the coordinates given on the table with respect to time (x) and height (y).
- What shape does the graph form? *parabola*
- Why, if the container is shot straight up and supposedly comes straight down, does the graph form this shape? *The graph represents time vs. height and is not the path of the container.*
- What is the maximum amount of time our superhero has to save the country? *102 seconds*
- Have the students take one approach or the other—either the container is intercepted or not—and write a conclusion to the scenario.
- Using the tables allows the students to see the x values with the corresponding y values; and with the tables, an understanding of the increasing and decreasing of the equation can be developed. However, although this is a good visual method of exploring the equation, finding exact values may be tedious, as in the case of finding the exact time the slime blob landed on Washington, D.C. Therefore, the problem will be solved by using the graphs of the quadratic, the zoom feature, and a system of equations approach to explore solutions.
- Using the problem $y = 100 + 500x - .5(9.8)x^2$, enter this into the <**Y=**> screen as done in the previous example.

Discussion: Once the equation is in place the x and y axis must be established to give an accurate picture of the graph. Do this by pressing <**WINDOW**>.

- What would be suitable for a minimum and maximum x?
- What would be suitable for a minimum and maximum y?
- Note: when using a large axis, it may be easier to set the x and y scale at 0.



- Now the equation is entered and the window is set. Press <**GRAPH**> and the equation will be graphed. The student can use <**TRACE**>, which would give an approximate maximum height, an approximate landing time, and approximate heights at different times. The <**ZOOM**> feature can be used to better pinpoint the maximum height and landing position (zero of the equation).

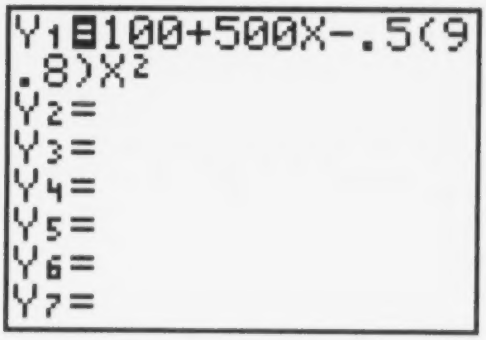


- The 1 in the top right corner represents the Y_1 equation. The X = and Y = represent the x and y coordinates at the starred point.

Finding the zero using the intersect method.

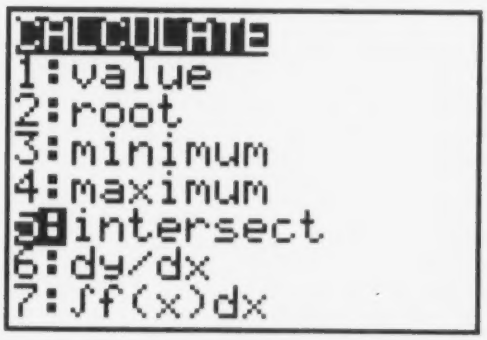
The following approach is a more efficient method of finding the zero of an equation.
To find the zeros of an equation, whether quadratic or of higher degree, the equation can be set up as a system, and use a feature of intersection or root.

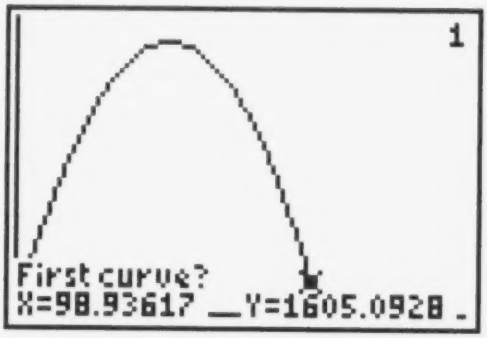
The zero of an equation is where the graph crosses the x-axis ($y = 0$).
Therefore, the system that will be used is where Y_1 = current equation and $Y_2 = 0$.
For example, with the equation $Y = 100 + 500x - .5(9.8)x^2$, enter in <**Y=**> as shown in the next illustration.



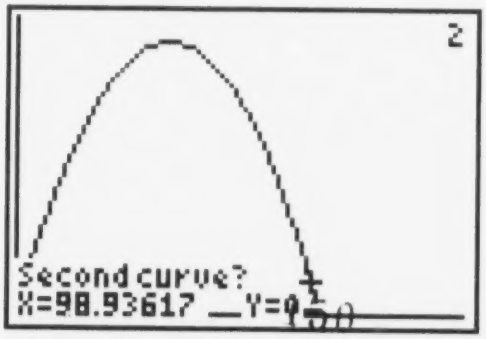
- Use the discussion from the previous problem to set the viewing window after pressing <**WINDOW**>.

- To find the zero:
  Press <**GRAPH**> to see the two equations graphed, then <**2nd**> <**TRACE**> and highlight the fifth option, which is intersect (Left screen). Then press <**ENTER**> (right screen below).

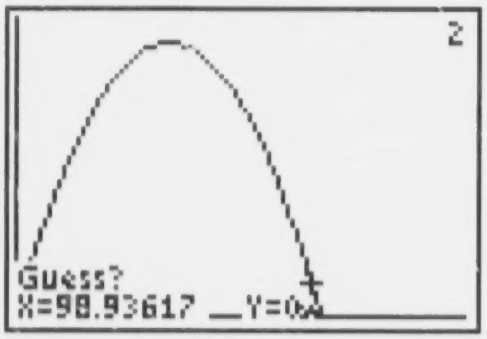




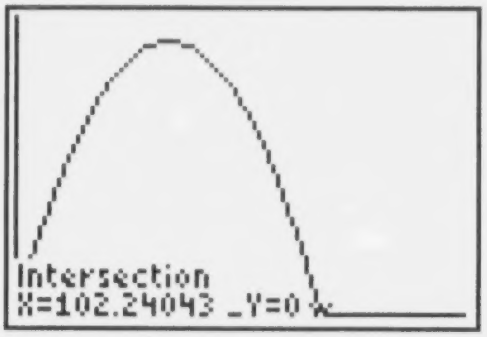
- The screen will display First curve?, which means the cursor is moving along the Y_1 equation. Use the arrow keys to move the cursor as close as possible to the intersection point (where Y = 0). Press <**ENTER**>.



- The screen displays Second curve?, which means the cursor is moving along Y_2 equation. Use the blue arrows to move the cursor as close as possible to the intersection point.
- Press <**ENTER**>.



- The screen displays Guess?, making the feature resemble a video game. Press <**ENTER**>.



- Notice when Y = 0, the *X*-value is 102.24043. This is a zero. If there is more than one zero, repeat the process beginning with <**2nd**> <**TRACE**>. When finding the other zeros, use the arrow keys to move your cursor to the next intersection point, and continue the process.
- Some of the stumbling blocks that could occur are discussed below.

Stumbling Block	Resolution
Calculator Freezes (will not proceed to second curve, guess, or intersection)	Press 2nd QUIT (above MODE) and start the process over
Y= may be in scientific notation	Press 2nd QUIT (above MODE) and start the process over or use the teachable moment and discuss notation

Practice problems

- Change the initial velocity on Handout 1 to:
  1. 1500 meters per second
  2. 3000 meters per second
  3. 5000 meters per second

- What effects does this have on maximum, minimum, and the times explored in the previous questions.

Enrichment

- Compare and contrast the solutions. Discuss the variations and have students write a synopsis of the activity.

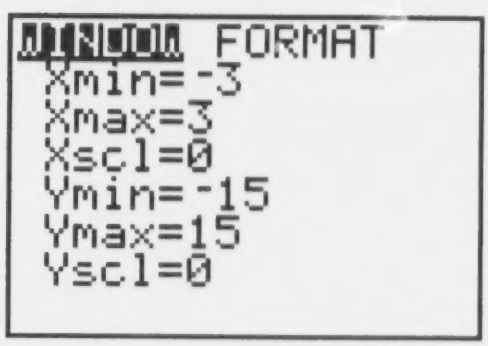
Teacher Notes for Handout 2

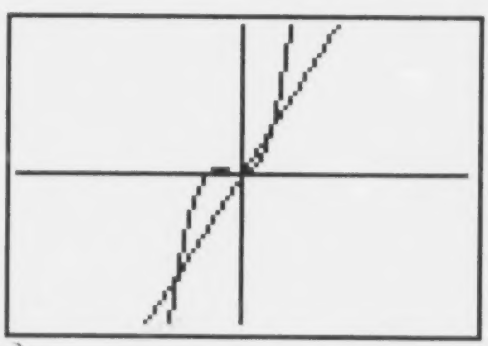
- Find the zeros of the following equations (include sketches of the graphs from your calculator).

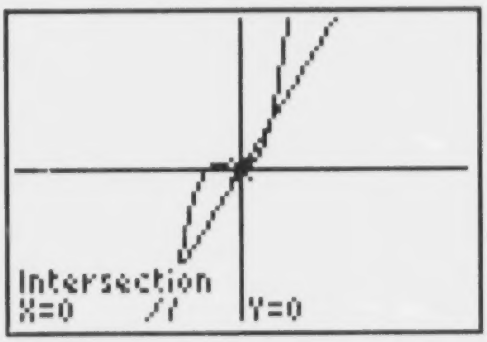
a. $35x^3 + 16x^2 = 12x$

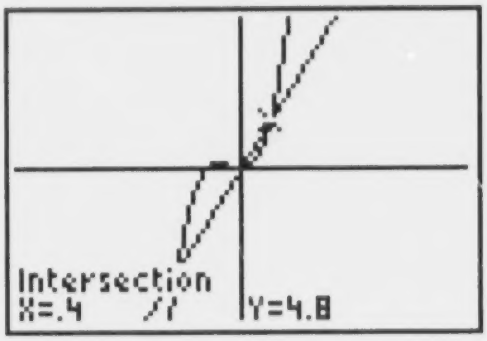
Intersect Method

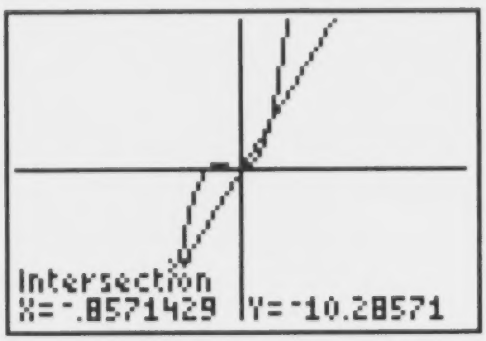
- One approach is to enter the left side of the above equation as Y_1, the right side of the equation as Y_2, and use **<2nd>** **<TRACE>** (CALC) to find the points of intersection (in this case, there are 3 points of intersection), which will find the values of x, which are the solutions of the equation.





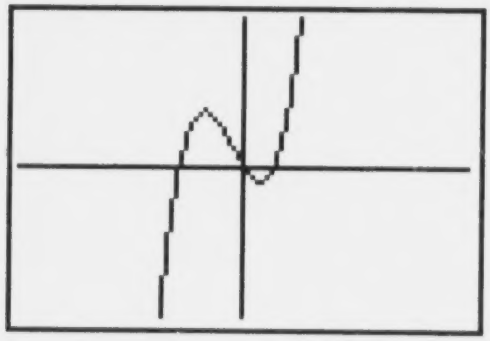






- The table can be checked to verify that these three values of x are zeros or approximate values for the zeros.

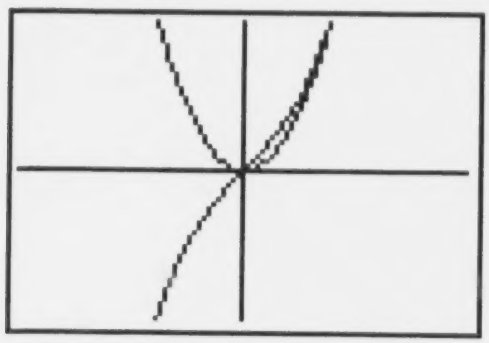
- These solutions are consistent with a graph of the original equation (rewritten with the right side equal to 0) and its intersection with $y = 0$. Then, with the left side as Y_1 and 0 as Y_2, the same zeros (0, .4, and -.8571429 [approximately]) can be found using <**2nd**> <**TRACE**> (CALC) three times with the cursor close, in turn, to each of the points of intersection.

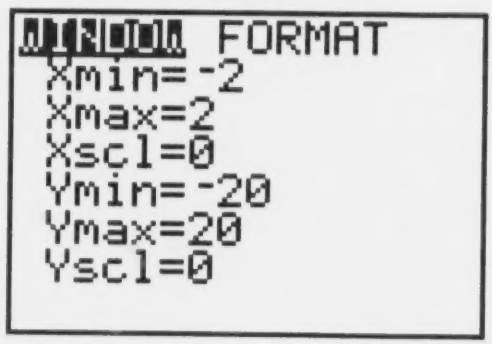


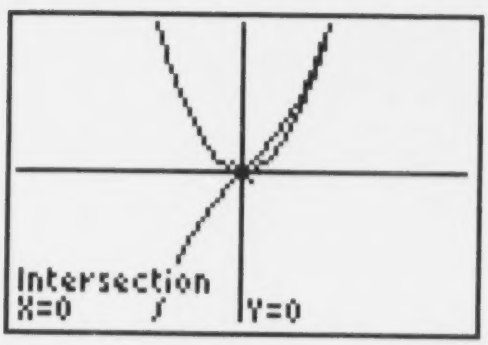
b. $18x^3 + 16x = 34x^2$

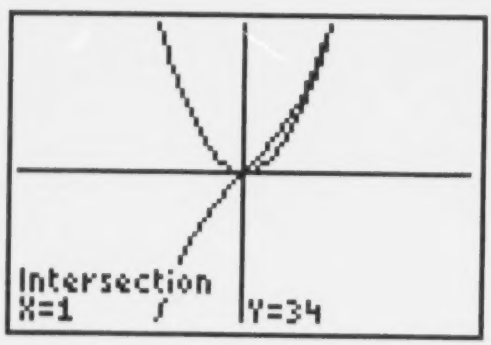
Intersect Method

- One approach is to enter the left side of the above equation as Y_1, the right side of the equation as Y_2, and use **<2nd>** **<TRACE>** (CALC) to find the two points of intersection, which will find the values of x, which are the solutions of the equation.

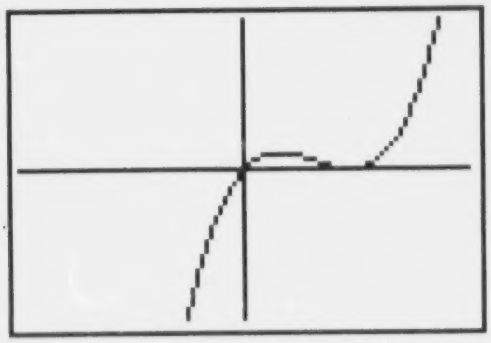






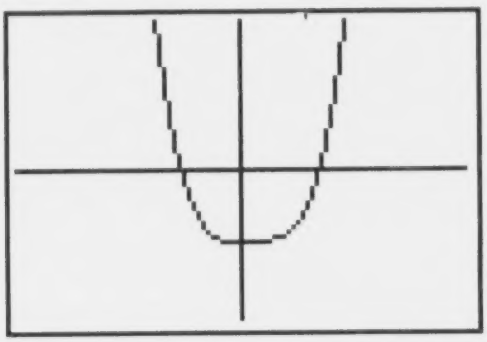


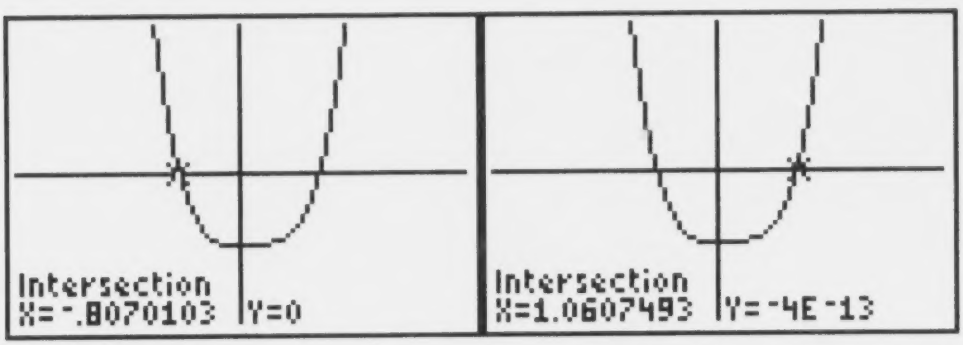
- Therefore, 0 and 1 are zeros of the original equation.
- Press **<2nd>** **<TABLE>** to check using the table method.
- Note: These solutions are consistent with a graph of the original equation as shown below, rewritten with the right side equal to 0 and its intersection with $y = 0$. Then with the left side as Y_1 and 0 as Y_2 the same zeros can be found.



c. $5x^4 - 3x^3 + 2x^2 - 5 = 0$

- Entering the left side as Y_1 and 0 as Y_2, the zeros (or approximate values for the zeros) can be found.





Adaptations:

- Have students create an equation with one real zero, two real zeros, or even four real zeros.
- What if the graph does not cross the x axis? What does this imply?

Activity One, Handout 1: Alien Attack

When an object is propelled upward, gravity pulls it back to Earth. The graph of time vs. height is a parabolic curve even though in a world in which the only force working on the object is gravity, the path of the object is straight up and down. The relationship of the object and gravity can be expressed by the formula:

$s = h_i + v_i t - 0.5gt^2$, where s is the distance above the starting point, h_i is the starting height, v_i is the initial velocity, t is time elapsed, and g is the acceleration of gravity. The acceleration of gravity is 9.8 meters/sec^2. Using this information, alternative methods of solving quadratics will be developed and practiced.

Problem:
An alien ship is hovering above Washington, D.C. at a height of 100 meters. A giant slime blob is housed in a missile-like container. The alien ship fires the container up into the air at a velocity of _____ meters per second, and sets their warp speed to fly away.

1. What is the height of the container at 25 seconds?

2. What is happening to the height after 25 seconds?

3. What is the maximum height the slime blob reaches?

4. After how many seconds does the slime blob reach its maximum height?

5. What is happening on either side of the vertex?

6. If a vertical line were drawn through the vertex what would it show?

7. What would be the equation of the vertical line?

8. What is happening to the slime blob on each side of the axis of symmetry?

9. Write the equation of the parabola formed in the form of $y = a(x - h)^2 + k$.

10. When the slime blob hits earth what is the ideal y-value?

11. After how many seconds would the slime blob slime Washington, D.C.?

12. What is the height when the slime blob is on the ground?

13. How close is .08325 to 0?

14. Make a graph with paper and pencil using the coordinates given on the table with respect to time (x) and height (y).

15. What shape does the graph form?

16. Why, if the container is shot straight up and supposedly comes straight down, does the graph form this shape?

17. What is the maximum amount of time a superhero has to save the country?

18. Decide whether the superhero saves the country, and create a conclusion to our scenario.

Activity One, Handout 2

- Find the zero's of the following equations (include sketches of the graphs from your calculator).

a. $35x^3 + 16x^2 = 12x$

b. $18x^3 + 16x = 34x^2$

c. $5x^4 - 3x^3 + 2x^2 - 5 = 0$

Inservice Suggestions:

- An inservice plan would be for the participants to perform as a class of high school students, following along with the instructor. The instructor's presentation would be greatly enhanced by using the view screen to illustrate the activities of the calculator.

Lesson 12: A Sick Astronaut

Authors

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Approximate Time Needed for Lesson

One traditional or one 90-minute class period

Content Area

Statistics and Probability; Estimation, Measurement, and Computation; Patterns, Functions, and Algebraic Concepts

Process Standards

Problem Solving, Communication, and Connections

Goal for Students

- To graph, interpret, and analyze data
- To apply concepts of slope and *y*-intercept to a problem situation
- To apply measures of central tendency in a problem situation
- To use the concept of line of best fit to describe relationship between two measures

Description or Summary of Lesson

Students in Arithmetic, Pre-Algebra, Algebra I, Algebra II, or Math Tech I and II are the intended audience. Students will investigate temperature data given in Celsius and Fahrenheit and use the graphing calculator to derive the conversion formula. Students will work in groups or individually. The lesson reviews linear equations and measures of central tendency by applying them in a motivational, problem-solving situation.

Background Information

Prior to this lesson, students will have studied slope, *y*-intercept, slope intercept form of equation of a line, coordinate graphing, and measures of central tendency.

Materials and Equipment

- View screen with TI-82 or TI-83 graphing calculator
- Overhead projector, if available
- Each student will need:
  - access to a graphing calculator
  - handouts

Activity: A Sick Astronaut

Teacher Notes

Students will use the graphing calculator to derive the conversion formula of Celsius to Fahrenheit: Fahrenheit = 9/5*Celsius + 32 or Fahrenheit = 1.8*Celsius +32.

Outline:

- Students, individually or in small groups, complete Handout 1 using either Handout 2 for the TI-82 or Handout 3 for the TI-83.
- Using the graphing calculator, students complete selected extensions from the list below.

The teacher may want to discuss with the students when the graphing calculator is preferred and when paper and pencil or mental calculation may be more efficient. For example, with only seven data points, the median and range and even the mean may be easier to calculate by hand. However, for large lists with many data points with several digits, the calculator may be more efficient. In any case, the graphing calculator method reinforces the concepts of range, mean, and median, as well as line of best fit, etc.

Extensions:

- Have students take their temperature every day in the morning and evening for one week and record their results in a table. At the end of the week, have them graph their data and look for relationships. They can find their mean temperature for each day—for the morning, for the evening, and for the entire week. They can find the median, mode, and range for these time periods as well. Have them bring their data back to the class and discover if the class has the highest temperature in the morning, evening, or on a particular day.

- Hang a thermometer in the classroom. Read and record the temperature every day at the same time. Find the measures of central tendency for the data. Graph the data. Is there any relationship between the temperature at one time in the day to another time?

- Suggestion: Assign students to research various temperature scales. Why is there more than one? Who invented each one? Why did they invent each scale? When? Have students write a brief report and include their sources.

- Complete a CBL activity such as those listed in the Suggested Teacher Resources that follow.

Suggested Teacher Resources:

Transforming Functions to Fit Data by Ronald J. Carlson and Mary Jean Winter, 1998, Key Curriculum Press. Activities: "Fire and Ice" and "Chill."

Real-World Math with the CBL System by Chris Brueningsen, Bill Bower, Linda Antinone, And Elisa Brueningsen, 1994, Texas Instruments. Activities: "Chill Out," "Mix It Up," and "And Now, the Weather."

CBL Explorations in Algebra for the TI-82 and TI-83 by the Meridian Group, 1996. Activities: "Exploring Statistics," "Exploring Exponential Equations," and "Exploring Scatter Plots."

CBL Explorations in Pre-calculus for the TI-82 by Meridian Creative Group, 1995. Activity: "Modeling Exponential Functions."

CBL Explorations in Calculus for the TI-82 by the Meridian Creative Group, 1995. Activity: "Newton's Law of Cooling."

Physical Science with the CBL by Donald L. Volz and Sandy Sapatka, 1997, Vernier Software. Activities: "Temperature Probe Response Time," "Boiling Temperature of Water," "Freezing and Melting of Water," "Evaporation of Alcohols," "Endothermic and Exothermic Reactions," "Neutralization Reactions," "Mixing Warm and Cold Water," "Heat of Fusion," "Energy Content of Fuels," "Energy Content of Foods," "Absorption of Radiant Energy," "An Insulated Coke Bottle," "A Good Sock," "Insulation Angle," and "Solar Homes and Heat Sinks."

Biology with CBL by Scott Holman and David Masterman, 1997, Vernier Software. Activities: "Energy in Food," "Biodiversity and Ecosystems," "Dissolved Oxygen in Water," "Watershed Testing," and "Physical Profile of a Lake."

TI-82 and CBL Lab Applications in Biology and Chemistry by Penny Sconzo and Chris Harrow, 1995, Kendall/Hunt Publishing Company. Activities: "Temperature vs. Volume of Gas," "Boiling Point Elevation," "Heat of Formation of Magnesium Oxide," and "Food Calorimetry."

Activity Handout 1: Sick Astronaut

Use a graphing calculator and the detailed instructions in Handout 2 (for the TI-82) or Handout 3 (for the TI-83) to help you answer questions related to the following information:

- Astronaut Iam Sick was ill for one week during training. The table shows his temperature on each day as measured by Nurse Careforu and Nurse Feelgood.

Day	Nurse Careforu	Nurse Feelgood
Sunday	38.3	101
Monday	39.2	102.5
Tuesday	38.2	100.8
Wednesday	38.6	101.4
Thursday	39	102.2
Friday	37.3	99.3
Saturday	37	98.6

1. With Nurse Careforu's temperature in degrees as the independent variable and Nurse Feelgood's temperature in degrees as the dependent variable, graph the information in a scatter plot. Fill in the Window you used and sketch the graph from the calculator screen.

Xmin	
Xmax	
Xscl	
Ymin	
Ymax	
Yscl	

a. The equation of the line of best fit is _____.

b. The slope is _____. Write the slope as a fraction._____.

c. The y-intercept is _____; thus, the point where the line intersects the y-axis is _____.

d. Interpret this point for Iam _____

e. Interpret it for the rest of the real world _____

f. Do you think there is a reason for the name of the nurses? If yes, explain. If not, reconsider what the nurses did in this situation.

_____

g. What is the standard formula for the relationship between degrees Celsius and Fahrenheit?_____ Where did you find this information or how did you calculate it?_____

h. How does the slope on the line of best fit and the y-intercept compare to the standard formula? _____

2. a. What was the range of Iam's temperature according to Nurse Careforu? _____

b. What was Iam's mean temperature for the week according to Nurse Careforu? _____ On which day(s) did this occur?_____

c. What was Iam's median temperature for the week according to Nurse Careforu? _____ On which day(s) did this occur?_____

d. What was the mode of Iam's temperature according to Nurse Careforu?_____ On which day(s) did this occur?_____

3. a. What was the range of Iam's temperature according to Nurse Feelgood?_____

b. What was Iam's mean temperature for the week according to Nurse Feelgood? _____ On which day(s) did this occur?_____

c. What was Iam's median temperature for the week according to Nurse Feelgood? _____ On which day(s) did this occur?_____

d. What was the mode of Iam's temperature according to Nurse Feelgood?_____ On which days did this occur?_____

4. Compare the results in #2 and #3. Explain what you just found.

Activity Handout 2: Sick Astronaut

TI-82 Instructions

- Turn the calculator on. Press <**2nd**> <**Y= >** (STAT PLOT). Press <**4**> (PlotsOff). This returns the cursor to the home screen.

- Press <**ENTER**>. This will turn all the stat plots off. Press <**Y=>. If** any of the = signs are highlighted, move the cursor on top of the = and press <**ENTER>.** Doing this will make the graph of that equation invisible. **Or** if you do not want that equation anymore, keep the cursor on top of the equation and press <**CLEAR>.** Do one of these two things for any equation that is in <**Y=>**.

- Press <**STAT**>. <**ENTER**> (Edit).

- In L_1 (List 1), list the temperatures taken by Nurse Careforu. After each number that is typed into the list, press <**ENTER**>. Press <▶> once.

- The cursor should now be under L_2 (List 2). Type the Temperatures taken by Nurse Feelgood into this list.

- Press <**2nd**> <**Y=>** (STAT PLOT) and <**ENTER**> (Plot 1). Turn the stat plot on by pressing <**ENTER**> while the cursor is flashing on top of On. Press <▼> once. Highlight the scatter plot and press <**ENTER**>. Press <▼> for Xlist and select (L_1).

Note: Remember L_1 is the list with temperatures taken by Nurse Careforu.

- Arrow down once to Ylist and select L_2.

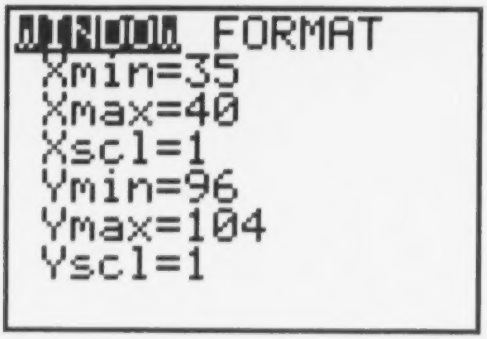
- Note: Remember that L_2 is the list of temperatures taken by Nurse Feelgood.

- Arrow down once and press <**ENTER**> with the cursor on the type of mark you want to represent the data.

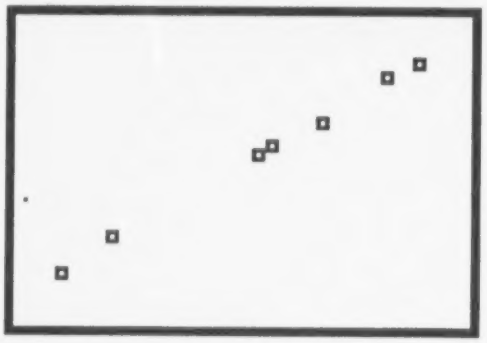
- Press <**WINDOW**> to set up the view screen where the graph will appear. Be sure to use numbers that include all of the data points. Buffer these points on all sides in order to see the complete graph.

- Xmin is the smallest value for the view screen horizontally. Use 35 for the Xmin. Arrow down once. The Xmax is the highest horizontal value for the viewscreen. Use 40 as the Xmax. Arrow down once. Xscl (*x*-axis scale) sets the tic marks on the axis. Type in 1 for Xscl because the range of the data is small. Arrow down once.

- The Ymin is the smallest vertical value of the viewscreen. Type in 96 for Ymin. Arrow down once. The Ymax is the highest vertical value of the viewscreen. Type in 104 for Ymax. Arrow down once. The Yscl (*y*-axis scale) sets the tic marks on the y-axis. Determine the scale number by the number used for the scale of the bar graph done by hand. The scale could also be determined by dividing the range of the *y* values by how many tic marks to be shown. Use 1 for the Yscl.



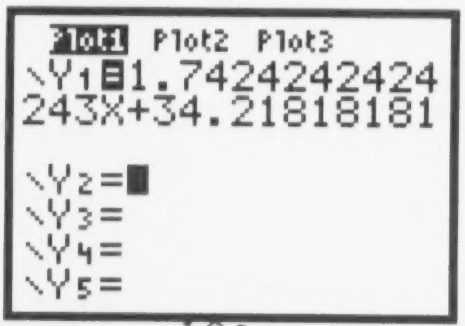
- Press **<GRAPH>**. The scatter plot will appear. (It should look similar to the scatter plot below.) To look at specific values, press **<TRACE>.** At the bottom of the screen, X= and Y= appear. Use the right arrow to move to points on the right and the left arrow to go back toward the first point.



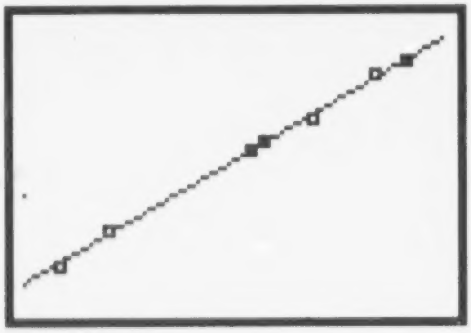
- To establish a line of best fit, press **<STAT>**. To calculate the equation of the line, press **<▶>**(CALC) **<5>** (LinReg(ax+b)). This will give the slope and *y*-intercept for the equation of the line of best fit. LinReg(ax+b) appears on the home screen. Press **<ENTER>**. Three values appear.

- The value for *a* is the slope and *b* is the *y*-intercept. The *r* value is called the correlation coefficient. The closer the line comes to hitting all of the points, the closer this number approaches 1. This *r* indicates a good fit of the line and the data points.

- Press **<Y=>.** To put this exact equation into the <Y=> menu, press **<VARS>**. The equation is coming from data, so press **<5>**. Then press **<▶>** **<▶>**, (EQ [equation]). Press **<7>** (RegEQ) to paste the equation into the **<Y=>**.



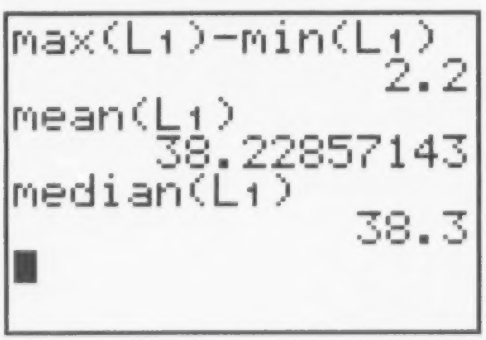
- The window is still set from earlier. Press <**GRAPH**>.



- The scatter plot will appear. (It should look similar to the scatter plot above.) To look at specific values, press <**TRACE**>. At the bottom of the screen, X= and Y= appear. Use the right arrow to move to points on the right and the left arrow to go back toward the first point.

- To determine the range for the temperatures taken by Nurse Careforu on the graphing calculator, press <**2nd**> <**STAT**> (LIST) and <▶> (MATH) <**2**> (max( ) <**2nd**> <**1**> (L_1) <) > < – >. Continue with <**2nd**> <**STAT**> (LIST) and <▶> (MATH) <**1**> (min( ) <**2nd**> <**1**> (L_1) <)>. Then press <**ENTER**> to get the range.

- Similarly, mean and media are 3: and 4: in the 2nd STAT MATH menu.

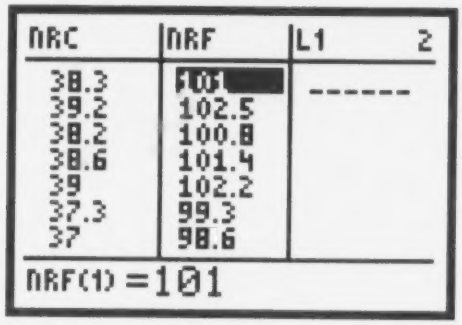


- Follow the same procedure as above, substituting <**2nd**> <**1**> <L_2> for the temperatures taken by Nurse Feelgood.

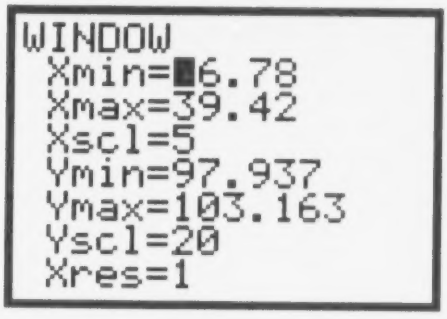
Handout 3

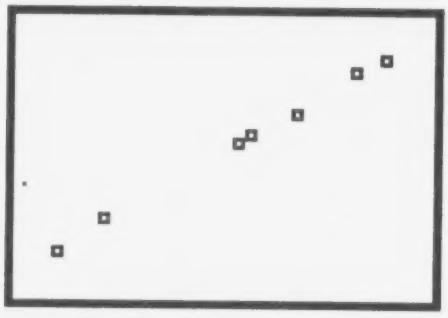
TI-83 Instructions

- 1. Enter each list of temperatures in a list labeled with the nurse's name, NRC or NRF, by pressing <**STAT**> <**1**> (Edit) and with the cursor on the top of a column, use <**2nd**> < **DEL**> (INS) and <**2nd**> <**ALPHA**> (A-Lock) and the keyboard to name a new list. Then arrow down to enter values.

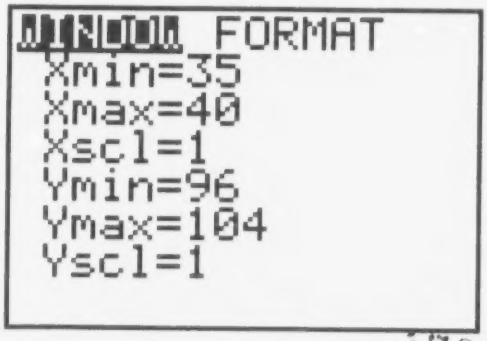


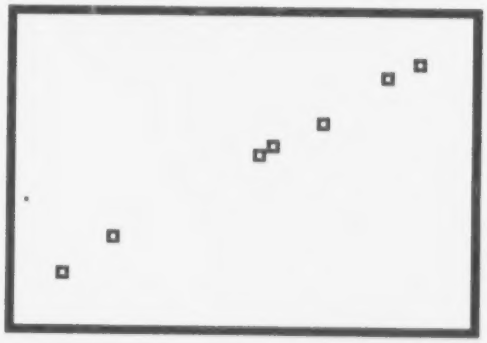
- Use <**2nd**> <**Y=**> (STAT PLOT) <**1**> <**ENTER**> and with On highlighted press <**ENTER**> and <**Down Arrow**> to select the scatter plot. Then <**Down Arrow**> and with <**2nd**> <**ALPHA**> (A-Lock) and the keyboard enter NRC for the Xlist and NRF for the Ylist. Also chose the first mark. Press <**WINDOW**> <**ZOOM**> <**9**> (ZoomStat) to see a graph of the data.





- Or press <**WINDOW**> to determine an appropriate window.





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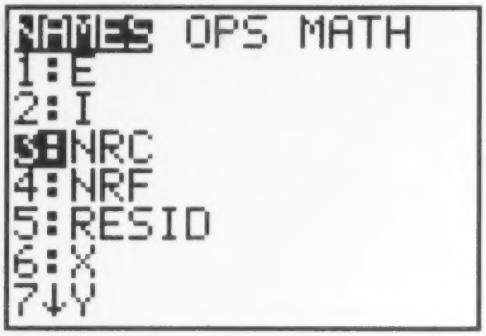
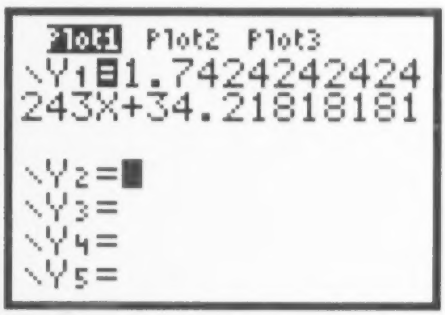
- Establish a line of best fit and write its equation by pressing <**STAT**> <▶> (**CALC**) <**4**> (LinReg(ax+b)) <**2nd**> <**STAT**>.

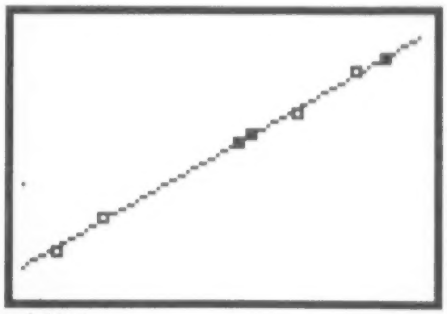


- Find NRC in the list of names and press <ENTER> then <,>. Again press <**2nd**> <**STAT**>, find NRF in the list of names and press <ENTER>

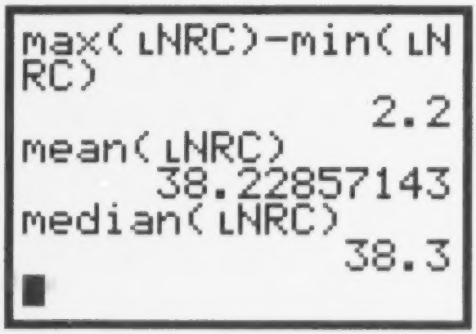
- Press <ENTER> and three values appear. The value for *a* is the slope and *b* is the *y*-intercept. The *r* value is called the correlation coefficient. The closer the line comes to hitting all of the points, the closer this number approaches 1. This *r* indicates a good fit of the line and the data points.

- Press <**Y=**> and clear Y_1. (Be sure that only the = sign for Y_1 is highlighted. Also, that all other plots are turned off under STAT PLOT 4:.) Press <**Vars**> <**5**> (Statistics) <▶> <▶> <**1**> to paste the equation of the line of best fit into the Y_1. Press <**GRAPH**> to see how well this line fits the data.





- To use the graphing calculator to determine range, press <**2nd**> <**STAT**> (LIST) <◆> <**2**> (max( ), <**2nd**> <**STAT**> (LIST), highlight NRC, and press <**ENTER**> <)> <–>.
- Next press <**2nd**> <**STAT**> (LIST) (Left Arrow> <**1**> (min( ), <**2nd**> <**STAT**> (LIST), highlight NRC, and press <**ENTER**>.
- This gives the range.
- Similarly, determine mean and median in turn using 3: and 4: in the 2nd STAT MATH menu.



- Repeat the above procedure with LNRC replaced by LNRF.

Lesson 13: Topographic Maps and Whitewater Rivers

Author

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Approximate Time Needed for Lesson

Two to five traditional class periods; also works well in 90-minute blocks; lesson also assumes that some of the work is completed outside of class as homework

Content Area

Estimation, Measurement, and Computation; Probability and Statistics; and Patterns, Functions, and Algebraic Thinking

Process Standards

Connections and Problem Solving

Goals for Students

- To gather and analyze data from a real-world phenomenon that can be modeled graphically
- To apply the concept of slope as a rate of change to a real-world situation

Description or Summary of Lesson

Advanced students in Algebra, Pre-Calculus, or Calculus are the intended audience. In this lesson, students work in groups of two or three to measure distances between contour lines on a topographic map, following the course of a river as it winds through a mountainous region. By converting these measurements to standard units such as feet, meters, or miles, and by using the elevation intervals between contour lines, students enter their data into lists that can be graphed as a scatter plot to depict the gradient profile of the river. In Algebra classes this information can enhance the concept of slope, and in more advanced classes the lesson can be used to lay the foundation for the concept of a derivative. The included sample data from a topographic map containing a steep creek can be used throughout this lesson; however, the lesson can be enhanced by using a topographical map of the local area.

Background Information

This lesson is an enrichment-level lesson or student project that allows students to apply previously learned concepts of measurement and slope in a real-world situation. The application can enhance their understanding of the concept of slope as rate of change. This lesson assumes prior lessons have given students experience in using the graphing calculator to analyze data in addition to proficiency with basic keystrokes.

Materials and Equipment

- View screen with graphing calculator and overhead projector, if available
- Each group of students will need:
  - √ access to a graphing calculator
  - √ a local area topographic map
  - √ a length of dental floss or fishing line
  - √ a ruler
  - √ handouts

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Activity One

Teacher Notes

Note: Though the Bear Creek data included in this activity can be used, teachers likely will find it more worthwhile to use topographic maps of local rivers and streams. These can generally be purchased inexpensively from outdoor shops and some businesses. In this case, students will need rulers that can measure or estimate lengths down to 1/10 or 1/16 of an inch. The transparent, flexible types are particularly useful. Also, the teacher will need to give some initial explanation as to how to read data from topographical maps.

If students are using the sample data included, skip Section I below and go to Section II of the teacher notes.

Section I: Reading Topographic Maps and Entering Data

If students are able to obtain actual topographic maps, select a stream or river that has several contour lines crossing it. Contour lines are shown as thin, brown lines that follow a wavy path or occur in groups and look like parallel bands or concentric patterns.

Contour lines will always close on themselves if traced with a pencil, even if you have to connect adjoining maps together to continue the line. Generally, the steeper streams and rivers give data that is easier to work with. Streams and rivers appear as blue lines on a map. Twenty to 50 contour lines give enough points. The spaces between the lines represent constant elevation changes. This separation distance is known as the *contour interval*, and will generally be given in 10-, 20-, or 40-foot intervals. The exact value for a particular map can be found at the bottom of the map.

The map will also have a scale that can be used to convert distances measured on the map to feet, meters, or miles. Many of the most common maps are of the 1:24,000 series. For example, 1/4 of an inch on the map corresponds to 24,000 x (1/4) inches on land, which is 6,000 inches or approximately .095 miles.

As mentioned earlier, topographic maps such as the one being used in this activity can be purchased for a moderate price from various outdoor shops and topographic agencies. Bear Creek is a particularly steep stream and is popular among expert kayakers and canoeists for its big drops and difficult rapids.

Students should select a convenient starting point on their stream and determine the elevation of that point. Selecting a point on the stream near a contour line makes it easier to find such a point. The task of data collection is described below.

> Students should measure, as accurately as possible, distances between successive contour lines as they cross the stream, putting these values in a list. If a TI-83 is being used, then students can give meaningful names to their

lists by highlighting a list name, such as L_1, pressing **<2nd> <DEL>** (INS) and entering an appropriate name to the list, such as FEET. There is nothing wrong, of course, with using any of the regular lists. For example, the first few measurements may be 0, 1", 1/2", 1/4", which indicate that starting with the first contour line (0 inches from itself), the second contour line came 1 inch later along the stream, the third line was 1/2 an inch from the second, the fourth line was 1/4 of an inch from the third, and so forth. It's important to measure the actual length of the stream, which may bend or contain turns, between consecutive contour lines. Using flexible material like dental floss or fishing line is useful, though many rulers are flexible and can bend. This process should continue until a convenient stopping point is found, or until the stream flattens out in a valley or plain.

- Go to Section III.

Section II: Entering Sample Data into a Graphing Calculator

- The starting point for the Bear Creek data, which we are using in this activity, begins at the 490-meter contour line (close to 1580 feet above sea level), at the place where the Scenic Highway crosses Bear Creek.
- On the Bear Creek map, 25 data points were collected, stopping in Sitton Gulf, at the 250-meter contour line.

DATA for BEAR CREEK

Distance Between Contours (Miles)	Distance From Start (Miles)	Elevation (Meters)	Elevation (Feet)
0.00	0.00	490	1607.69
0.28	0.28	480	1574.88
0.28	0.56	470	1542.07
0.25	0.81	460	1509.26
0.11	0.92	450	1476.45
0.17	1.09	440	1443.64
0.10	1.19	430	1410.83
0.06	1.25	420	1378.02
0.05	1.30	410	1345.21
0.05	1.35	400	1312.40
0.04	1.39	390	1279.59
0.10	1.49	380	1246.78
0.08	1.57	370	1213.97
0.08	1.65	360	1181.16
0.09	1.74	350	1148.35
0.11	1.85	340	1115.54
0.13	1.98	330	1082.73
0.10	2.08	320	1049.92
0.10	2.18	310	1017.11
0.10	2.28	300	984.30
0.13	2.41	290	951.49
0.12	2.53	280	918.68
0.11	2.64	270	885.87
0.21	2.85	260	853.06
0.39	3.24	250	820.25

- If a TI-83 is being used, then students can give meaningful names to their lists by highlighting a List name, pressing <**2nd**> <**DEL**> (INS) and giving an appropriate name to the list. There is nothing wrong, of course, with using any of the regular lists.

- Enter the Distance Between Contours data into L_1 or an otherwise named list.

- Continue with Section III.

Section III: Constructing the Scatter Plot:

- The next task is to get a cumulative sum of the distances, so that one knows the distances of consecutive contour lines from the starting point. Students will discover all kinds of clever ways to accomplish this.

- One way to do this on the TI-83 if the original data was listed in L_1, is to highlight L_2, and press <**2nd**> <**STAT**> (LIST) <**▶**> (OPS) <**6**> (cumSum( ) and then press <**2nd**> <**1**> (L_1) <)>.

- The accumulated distances will be recorded in L_2. It may be useful to name this list *DIST* if you are using a TI-83. These distances should then be converted to miles, or km. Remember that each inch corresponds to 24,000 inches, (2,000 ft.), if you are using one of the standard 24 000 series topographic maps. Highlight L_2 (or *DIST*) and then multiply by 24,000/(12 x 5280) to convert inches to miles on these maps.

- L_2=24000* L_2 at the bottom of the screen and <**ENTER**> will place the appropriate calculations in L_2. Similar calculations can be made if metric units are being used.

- The final step in preparing the scatter plot is to associate each of these accumulated distances with the elevation of that contour. Again, allow students to devise ways to do this on their own. Some will no doubt just laboriously enter each of the values in L_3 (or *ELEV*), but others will recognize that since the contour lines occur at regular intervals (say 20 feet or 10 meters), a sequence can be used to quickly list the values. The particular map which we are using in this activity uses 10-meter contour intervals. The following sequence can be used to enter the values.

- Highlight L_3 (or label a list as *ELEV*), and press <**2nd**> <**STAT**> (LIST) and with OPS highlighted press <**5**> (seq( ) and then complete seq(490 - X, X, 0, 240, 10). Other sequences can also be used.

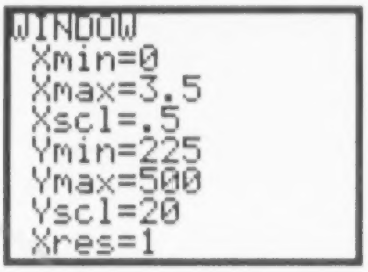
- Students are now ready to construct a scatter plot of their data. For the data collected in Section I or the Bear Creek data in Section II, Plot 1 is turned on, using the second of the Type options (which draws a line segment between consecutive data points), with DIST or L_2 in Xlist, and ELEV or L_3 in Ylist.

TI-83

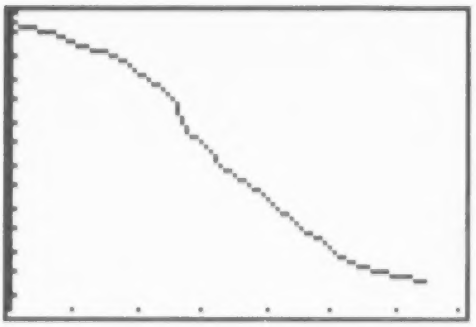
TI-82

- It is probably more useful to have students set their own windows by observing the values they recorded than to allow them to simply use the **<ZOOM>** **<9>** (ZoomStat) option.

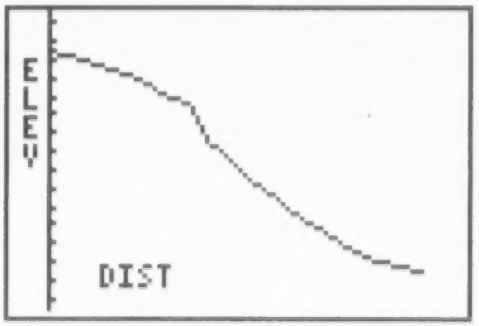
- In this way, they will realize that the Xmin and Xmax are associated with distances along the stream from the starting point, and that the Ymin and Ymax values are associated with the ending and starting elevations. The Xscl and Yscl can be set as desired. On the Bear Creek data, Xscl was set at .5 miles and the Yscl at 20 meters.



- A graph similar to the following should be seen.



TI-82



TI-83

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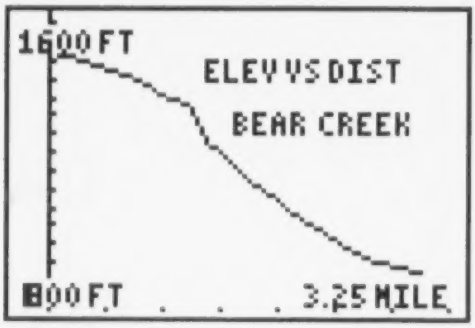
- Students who are familiar with using *Graph Link* to print their graphs can make their graphs more interesting by writing useful information on the scatter plot using the **<2nd>** **<PRGM>** (DRAW) **<0>** (Text( ) option.



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Handout Two Answers (for Bear Creek)

1. The average gradient is -243.04 foot/mile, or about -45.93 m/km, which means that the creek is dropping, on the average, about 243.04 feet for every mile it travels. The negative sign reflects the fact that rivers run down hill. If the entire river were tilted down at a constant level of 243.04 vertical feet for each horizontal mile traveled, then the gradient would be constant. Of course, most rivers contain bends, rapids, fast and slow places, so a constant gradient is not very realistic.

2. The gradient is a measure of vertical drop in a river over a horizontal distance, which can be expressed as the basic definition of the slope of a line between two points.

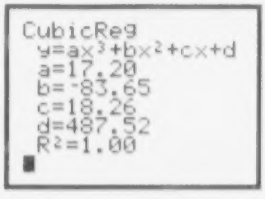
   Gradient = change in elevation / elapsed distance.

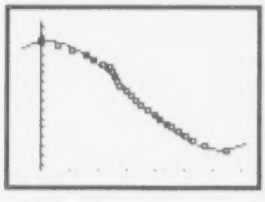
   Calculus students may recognize that if these *difference quotients* were calculated over smaller and smaller distances, then they would be finding approximations for the exact rate at which the river is changing at that point, which is related to the concept of the derivative.

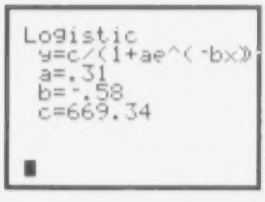
3/4. The river is steepest where the gradient is the largest, which occurs in the middle of Bear Creek. Between the 1.2 and 1.65 mile marks, Bear Creek is dropping at an incredible rate of just over 500 ft. per mile. Expert kayakers call this section the Cosmic Mile, as it contains some of the steepest and most intense whitewater rapids in the country.

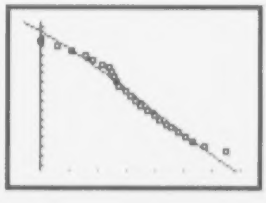
5. The riverbed is likely to be the most resistant to change at the beginning and end of Bear Creek, as the harder rock strata in these places are not as easily cut away by the moving water. As a result the gradient is less, and the scatter plot is flatter, in these areas.

Extra Credit: This is primarily for more advanced students. Those students who are familiar with various curve fitting, or regression techniques, will find that a cubic or a logistic function will fit the data well, though it is useful for students who know how to do so, to check the residuals of their equations







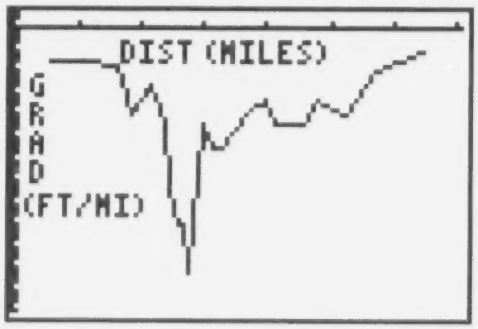


Extensions:

1. It is interesting for students to calculate the gradient (slope) of the segments between consecutive contours. These are *difference quotients*. If these values are then plotted against the midpoints of the intervals, one can observe the rate at which the gradient of the river is changing. Since the contour interval is fixed (10 m $\approx$ 32.81 feet on Bear Creek), then one can calculate the difference quotient by dividing 32.81 feet by the length of the river segment between those two contours. An easy way to do this is to divide 32.81 feet by the elements in Column 1 of the previous table. Doing this will result in the following gradients between each consecutive pair of contour lines on Bear Creek: units are in feet per mile.

0.00
117.18
117.18
131.24
298.27
193.00
328.10
546.83
656.20
656.20
820.25
328.10
410.13
410.13
364.56
298.27
252.38
328.10
328.10
328.10
252.38
273.42
298.27
156.24
84.13

2. A scatter plot of the rates of change calculated in the previous activity vs. distance gives the graph below. Note that students may wish to use the midpoint of the segments between contours, or work with one of the end points of the segment. Calculation of the



midpoints is a good exercise in its own right. This graph uses the right end points. Question: What do the spikes on this graph indicate?

Answer: The spikes represent particularly steep sections on the river, where large drops and rapids are likely to be found. The particularly big spike in this graph occurs on a section of the river that includes a large, 65-foot, 3-tiered waterfall known by expert kayakers as Stairway to Heaven.

3. If possible, have your students hike along segments of their river or stream, so that they can see what the river is actually like, and compare what they see to what they predicted.

4. Using highways instead of rivers gives an opportunity to have both increasing and decreasing components on the scatter plot, since roads go both up- and downhill.

5. Another extension is to have students create a topographic map with specified features, which in turn they can analyze. An example follows. Several solutions are possible. It is productive to have the students work in groups and discuss their maps with the rest of the class.

- Create a map of a river on a 1:24 000 scale, using 20-foot contour lines, with the following characteristics:

1. The river is 100,000 feet long.

2. The river begins on a relatively flat plateau, falls through a relatively mild canyon, enters an area where the rock is hard and the river doesn't cut through the ground as fast as in the canyon. It then enters a deep gorge where the river drops at a relatively fast rate until it finally enters a valley where it slows before emptying into a large lake.

3. The river drops 1,000 vertical feet in elevation.

4. Sketch the river as realistically as you can, including contour lines that cut across the path of the river.

Suggestions for Assessment:

- Each of these categories will be assigned points deemed appropriate by the instructor. Points can vary within each subcategory, depending on the instructor's discretion.

- A paper discussing the group's findings and a group presentation to the rest of the class are important parts of the activity. The following is a sample of one assessment rubric.

1. Remember that a significant amount of time is used by the students in gathering data.
   20 points:
   - √ up to 10 points for participating in the data collection and supporting a collaborative effort
   - √ up to 5 additional points for collecting the necessary data
   - √ up to 5 additional points for accuracy and clarity

2. Report on the activity
   50 points:
   - √ 10 points: Explanation of the various procedures used in the data collection, and analysis. Attention should be paid to how well students explained how they used technology in their work.
   - √ 10 points: correctly entering data into tables and lists
   - √ 10 points: obtaining good scatter plots
   - √ 20 points: Discussion of the analytical features, using tables and graphs to illustrate their conclusions. Students should be encouraged to use Graph Link printouts.

3. Class Presentation
   20 points:
   - √ 5 points for being prepared and ready to present on time
   - √ 10 points for a clear explanation of their findings
   - √ 5 points for participation by all group members

4. Activity Handout Two
   10 points:
   - √ 5 points for correctness
   - √ 5 points for contributions by all group members

Handout 1

Student Instructions:

- You will be assigned partners by your instructor and will be working with your group for several sessions. You need to bring your graphing calculator, a ruler, pencil and paper, and a small length of dental floss or fishing line to the sessions.

- Your group is to decide on a stream or river you would like to study. For the purposes of this activity, a steep stream or river works best. Your group will procure a topographic map that includes the river you wish to study. Your instructor can suggest places where you can find topographic maps, and give you some basic instructions on how to gather information from your map.

You are now ready to begin work.

1. Select a convenient starting and stopping place on your river. Select segments of your river that are crossed by 20 to 50 contour lines.

2. Record the elevation of your starting and ending locations. Be sure to include the correct units.

3. This is the most tedious part of the project, but the most critical part for accuracy. It's easy to get lost in regions of tightly packed contour lines, so be careful not to lose your place.

   Beginning at your starting point, measure the distance to the next contour line that crosses the stream in fractions of an inch or centimeter, and record this value. From this second contour line, measure the distance to the next contour line. Continue in this fashion, recording each successive distance. In some places your river may bend or contain turns. Use the dental floss to help you measure these distances.

4. Using the scale at the bottom of your map, convert the distances in #3 to miles (or km). Enter these values in a list on your calculator.

5. Use the values in #4 to find how far each successive contour line is from the starting point. If you are using a TI-83, you can use the cumSum(feature to help you find these values. If you are using a TI-82, you can add each successive number to the previous sum. Enter these values in L_2. If you are using a TI-83, you may wish to name your list *DIST*.

6. Remembering that the contour lines are separated by a fixed value (the contour interval), you can determine the elevation of each of the contour lines that you measured. Record these elevations in L_1, or name it *ELEV*.

7. You are now ready to set up your scatter plot of L_3 vs. L_2 (*ELEV* vs. *DIST*). Set an appropriate window based on the values you have recorded in the two lists. It will be useful to use the connected type of scatter plot.

8. Answer the questions on Handout Two.

9. Submit a well-written paper that discusses your work and your results on this project. Explain what you did to collect your data and how you analyzed your results. Clearly explain how you arrived at any calculated values. Graphs, tables, and data should all be presented in a satisfactory manner. Explain how you used technology in your work. What have you discovered about the nature of the stream you chose? As always, you are expected to use complete sentences with good grammatical form and pay attention to spelling.

10. Your group should prepare a 10- to 15-minute presentation of your findings for the class. Each person in your group must participate in the presentation.

Extra credit:

- Additional information about your stream can be learned by calculating the slopes between consecutive contour lines and constructing a scatter plot of these values on the y-axis, and their distances from the starting point on the x-axis. You may wish to find the midpoints of the segments between the contour lines, or use right or left end points.

- Explain what you did and tell what additional information this gives you about your stream.

Handout 2

- Using the data in Handout 3, answer the following questions as completely as possible. Write in complete sentences and pay attention to good grammatical construction and correct spelling. Show how you arrived at any of your calculated values.

1. The *gradient* of a section of river can be thought of as the slope of the line between the starting and ending points for that segment of the river. What is the average gradient of your stream? What does the sign of this number mean? What do your units mean? What would your river look like if the gradient were constant over its entire length? Is this realistic?

2. Explain in words how the *gradient* is a rate of change. Express this as a mathematical formula.

3. Over which sections is your river the steepest? How steep is the river at these places?

4. Where do you expect to find the most severe rapids or falls in your stream? Why?

5. Where on your river will the riverbed contain material that is most resistant to change?

Extra Credit:

- Can you find an equation, or mathematical function, that approximates the gradient profile of your river?

Handout 3

DATA for BEAR CREEK

Distance Between Contours (Miles)	Distance From Start (Miles)	Elevation (Meters)	Elevation (Feet)
0.00	0.00	490	1607.69
0.28	0.28	480	1574.88
0.28	0.56	470	1542.07
0.25	0.81	460	1509.26
0.11	0.92	450	1476.45
0.17	1.09	440	1443.64
0.10	1.19	430	1410.83
0.06	1.25	420	1378.02
0.05	1.30	410	1345.21
0.05	1.35	400	1312.40
0.04	1.39	390	1279.59
0.10	1.49	380	1246.78
0.08	1.57	370	1213.97
0.08	1.65	360	1181.16
0.09	1.74	350	1148.35
0.11	1.85	340	1115.54
0.13	1.98	330	1082.73
0.10	2.08	320	1049.92
0.10	2.18	310	1017.11
0.10	2.28	300	984.30
0.13	2.41	290	951.49
0.12	2.53	280	918.68
0.11	2.64	270	885.87
0.21	2.85	260	853.06
0.39	3.24	250	820.25

Lesson 14: We've Got the Spirit—A Variety of Techniques for Solving Linear Systems

Authors

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Approximate Time Needed For Lesson
One to two traditional class periods or one 90-minute period

Content Area
Patterns, Functions, and Algebraic Thinking

Process Standards
Problem Solving, Reasoning, and Connections

Goals for Students

- To solve linear systems using a variety of techniques

Description or Summary of Lesson

Students in Algebra I or Algebra II are the intended audience. However, while two-variable systems may be used in Algebra I or II, linear programming and three-variable systems are usually more appropriate for Algebra II students. This lesson is problem centered and applies various solution methods involving systems of equations and inequalities for problems written to be of interest to high school students. This lesson is designed to be either teacher-directed or as a collection of activities that may be completed in cooperative groups, for students with adequate experience with the graphing calculator.

Background Information

Prior to this lesson students will have solved linear equations and inequalities in one variable and graphed linear functions. They will also have solved systems of equations algebraically and graphically without graphing calculators.

Equipment and Materials

- View screen with TI-83 graphing calculator
- Overhead projector
- Each student will need:
  - √ access to a TI-83 graphing calculator
  - √ copy of each handout

Activity One: A Homecoming Promotion I

Teacher Notes

Using an appropriate domain needs to be emphasized in every problem. When using the calculator for graphical solutions, students become frustrated when they do not see the expected graph in the viewing window. In each problem it is always important to emphasize checking for the reasonableness of answers given the "real-world" situation. Students should be able to justify rounding or not according to the situation. Variables must always be clearly identified by the students at the outset and in each succeeding problem or extension.

Two-Variable Systems

- The Spirit Club has $200 to spend on souvenir items to be given away at "Homecoming." They would like to buy enough items so the first 500 fans who arrive receive either a football or a spirit ribbon. The footballs, which are more popular, cost 60 cents each; the spirit ribbons, 25 cents each. Keeping all of these facts in mind, how many of each can they buy and get as close as possible to $200 without going over?

Algebraic Solution

- Let x = number of footballs; y = number of ribbons

$x + y = 500$
$.6x + .25y = 200$

$y = 500 - x$
$.6x + .25(500 - x) = 200$
$.6x + 125 - .25x = 200$
$.35x = 75$
$x = 214.3$

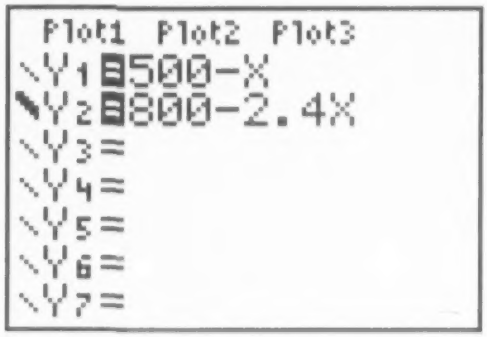
$y = 500 - 214.3$
$y = 285.7$

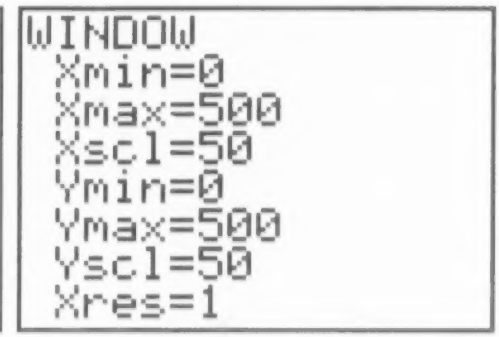
- Therefore, the students should purchase 214 footballs and 286 ribbons at a cost of $199.90.

Graphical Solution

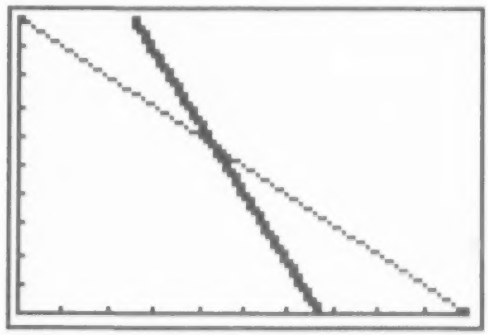
- Enter each equation into the Y= menu after pressing <**Y=**>. Use <left arrow> and <**ENTER**> keys to set one line to be thin and one to be thick.

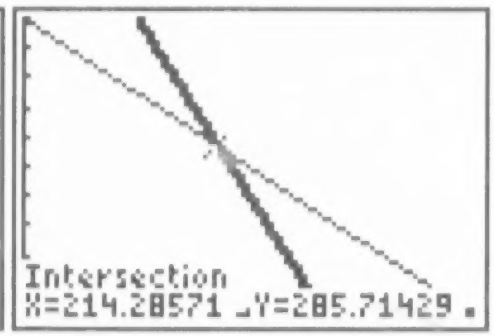
- Press <**WINDOW**> and determine an appropriate viewing window and scale. Because the students are purchasing items for 500 fans, both x and y will be between 0 and 500.





- To calculate the point of intersection, press <**GRAPH**> and then <**2nd**> <**TRACE**> (CALC).
- Next press <**5**> (intersect) and then <**ENTER**> < **ENTER**> < **ENTER**>.

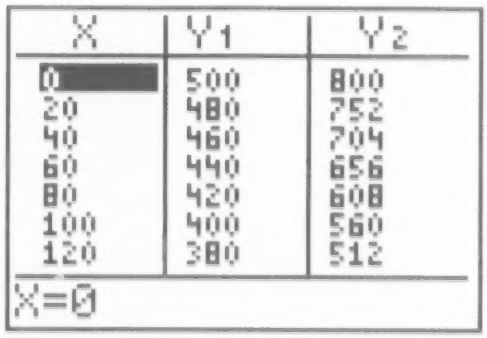




- Therefore, the students should purchase 214 footballs and 286 ribbons at a cost of $199.90.

Numerical Solution

- Using the functions in the <**Y=**> menu, examine the table of values in order to find the value of x (the number of footballs) for which y (the number of ribbons) is the same in both Y_1 and Y_2. <**2nd**> <**WINDOW**> (TBLSET) allows us to enter a starting value for x of 0 and an increment of 20. <**2nd**> <**GRAPH**> (TABLE) allows us to view values of x and y for each of the functions.



X	Y1	Y2
0	500	800
20	480	752
40	460	704
60	440	656
80	420	608
100	400	560
120	380	512

X=0

- Scroll through x values until the difference in Y_1 and Y_2 is the smallest.

X	Y1	Y2
160	340	416
180	320	368
200	300	320
220	280	272
240	260	224
260	240	176
280	220	128

X=220

- Then press **<2nd>** **<WINDOW >** (TBLSET) and adjust TABLE SETUP accordingly.

TABLE SETUP
TblStart=210
△Tbl=1
Indpnt: Auto Ask
Depend: Auto Ask

X	Y1	Y2
210	290	296
211	289	293.6
212	288	291.2
213	287	288.8
214	286	286.4
215	285	284
216	284	281.6

X=214

- The values of y are closest when $x = 214$; therefore, the students should buy 214 footballs and 286 ribbons at a cost of $199.90.

Extensions

A. Due to inflation the price of ribbons skyrocketed to 30 cents. How many of each should they buy now?

Solving the adjusted system of equations, $y = 166.666...$ and $x = 333.333...$ However, 167 of the footballs and 333 of the ribbons would total $200.10. If going over $200 is not allowed and if the Spirit Club must have 500 items, the 166 footballs and 334 ribbons should be purchased for $199.80.

B. A $10 rebate is given for every 200 footballs purchased. Using the inflated price for ribbons and keeping the total items to 500, how many of the popular footballs can now be purchased?

200 footballs and 300 ribbons could be purchased for $210.00 (includes the $10.00 rebate) for purchasing 200 footballs. This keeps the total items to give out at 500 but increases the number of the more popular footballs. The class could discuss other parameters and the corresponding solutions. 130

Activity One Handout

- The Spirit Club has $200 to spend on souvenir items to be given away at "Homecoming." They would like to buy enough items so the first 500 fans who arrive receive either a football or a spirit ribbon. The footballs, which are more popular, cost 60 cents each; the spirit ribbons, 25 cents each. How many of each can they buy and get as close as possible to $200 without going over?

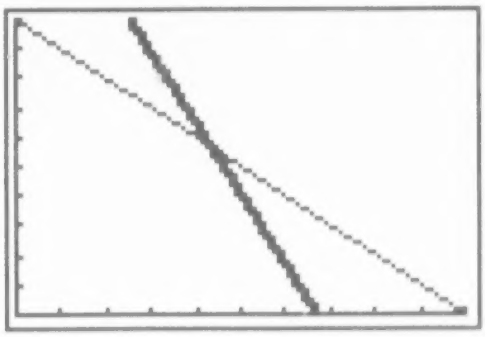
1. Solve algebraically below using a system of two equations with two variables each.

2. Follow the instructions below to solve graphically using your graphing calculator.

- Enter each equation into the Y= menu after pressing <**Y=**>. Use <left arrow> and <**ENTER**> keys to set one line to be thin and one to be thick.

- Press <**WINDOW**> and determine an appropriate viewing window and scale. Because the students are purchasing items for 500 fans, both x and y will be between 0 and 500.

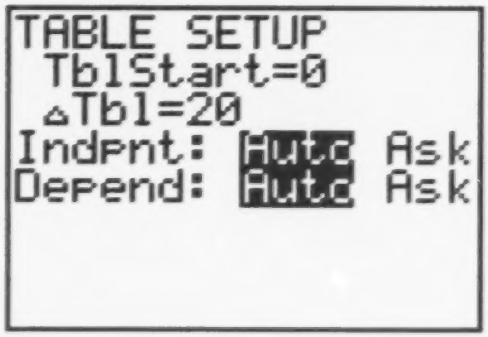
- To calculate the point of intersection, press <**GRAPH**> and then <**2nd**> <**TRACE**> (CALC).
- Next press <**5**> (intersect) and then <**ENTER**> < **ENTER**> < **ENTER**>.



- The point of intersection is _____.

- Therefore, the club should purchase _____ footballs and _____ ribbons at a cost of $_____.

3. Using the functions in the Y= menu, examine the table of values in order to find the value of x (the number of footballs) for which y (the number of ribbons) is the same in both Y_1 and Y_2. <**2nd**> <**WINDOW**> (TABLSET) allows us to enter a starting value for x of 0 and an increment of 20. <**2nd**><**GRAPH**> (TABLE) allows us to view values of x and y for each of the functions.



X	Y1	Y2
0	500	800
20	480	752
40	460	704
60	440	656
80	420	608
100	400	560
120	380	512

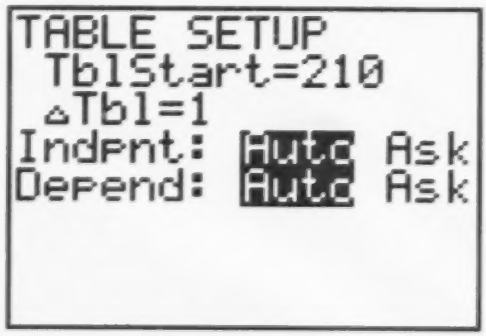
X=0

- Scroll through x values until the difference in Y_1 and Y_2 is the smallest.

X	Y1	Y2
160	340	416
180	320	368
200	300	320
220	280	272
240	260	224
260	240	176
280	220	128

X=220

- Then press **<2nd>** **<WINDOW>** and adjust TABLE SETUP accordingly.



- The values of y are chosen when x = _____; therefore, the club should buy _____ footballs

  and _____ ribbons at a cost of $_____.

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Extensions:

A. Due to inflation the price of ribbons skyrocketed to 30 cents. How many of each should they buy now? Determine and write below the appropriate system of equations. Then solve graphically on the graphing calculator and sketch the graph below. Finally, solve numerically using the table method and draw the final table screen below.

B. A $10 rebate is given for every 200 footballs purchased. Using the inflated price for ribbons and keeping the total items to 500, how many of the popular footballs can now be purchased?

Activity Two: A Homecoming Promotion II

Teacher Notes

Linear Programming

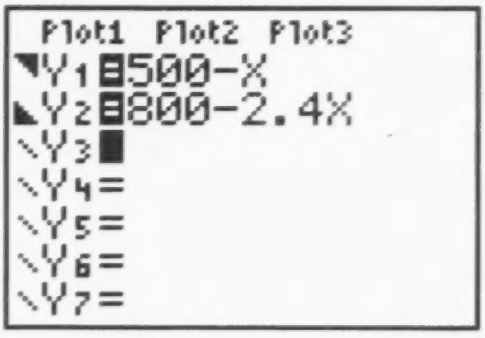
Using the basic problem from Activity One, suppose the students want to distribute at least 175 footballs. How many footballs and ribbons should be purchased to stay within the budget and still accommodate at least as many fans as before?

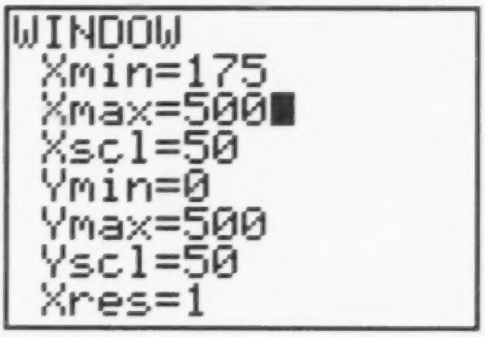
Linear Programming Solution

- Write the constraints for the system.

$x \geq 175 \quad (y \geq 0)$
$y \geq 500 - x$
$y \leq 800 - 2.4x$

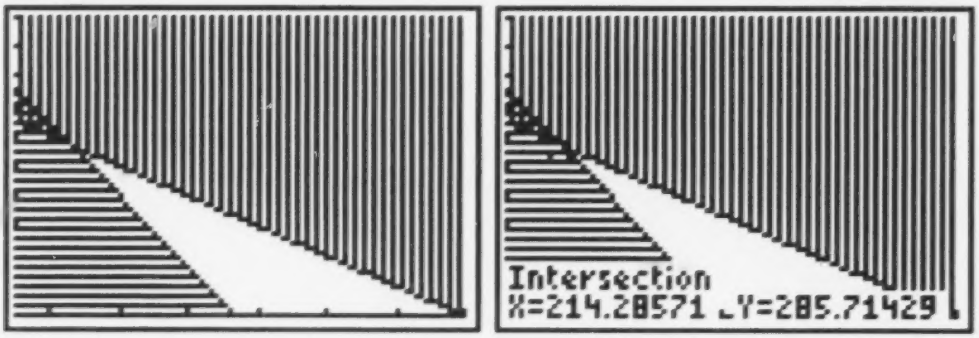
- Enter the equations into the <**Y=**> menu, select style, and determine an appropriate viewing window and scale. (Review Activity One for the steps.)





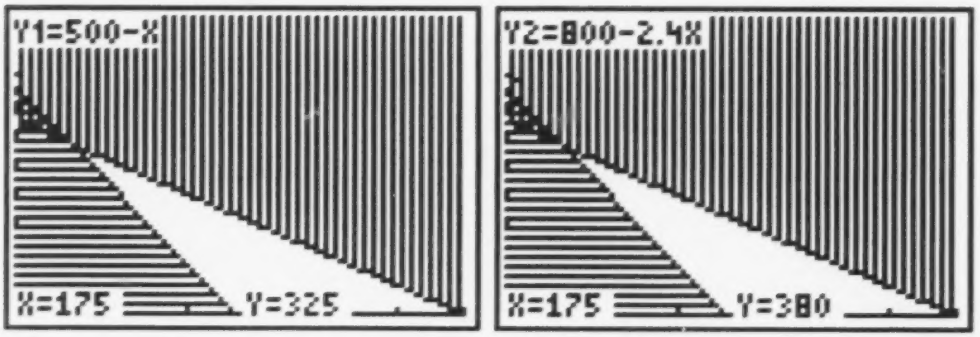
- Identify the feasible region and its corner points. To calculate the intersection of Y_1 and Y_2 press <**GRAPH**> and then <**2nd**> <**TRACE**> (CALC).

- Next press <**5**> (intersect) and then <**ENTER**> < **ENTER**> < **ENTER**>.



- The intersection of the two *Y* functions with the line $x = 175$ must be done by pressing <**2nd**> <**TRACE**> (CALC) <**1**> (value). Then type 175 and press <**ENTER**>.

- <▽> will give the second equation intersection.



- Thus, the corner points of the feasible region are (175,325), (175,380), and (214,285).

- Minimize the cost.

  C(x , y) = .6x + .25y
  C(175,325) = .6(175) + .25(325) = $186.25 *
  C(175,380) = .6(175) + .25(380) = $200.00
  C(214,285) = .6(214) + .25(285) = $199.65

- Maximize the service.

  N(x , y) = x + y
  N(175,325) = 175 + 325 = 500
  N(175,380) = 175 + 380 = 555 *
  N(214,285) = 214 + 285 = 499

- Therefore, to minimize cost, students should buy 175 footballs and 325 ribbons. To maximize service, students should buy 175 footballs and 380 ribbons.

Activity Two Handout

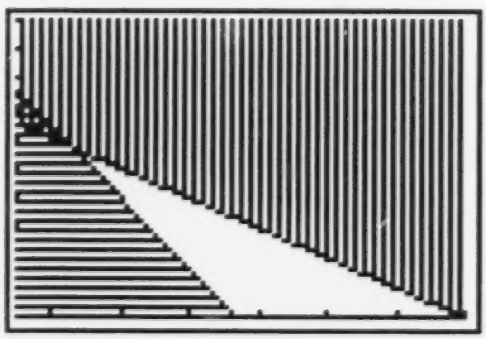
Using the basic problem from Activity One, suppose the Spirit Club wants to distribute at least 175 footballs. How many footballs and ribbons should be purchased to stay within the budget and still accommodate at least as many fans as before?

Linear Programming Solution

1. Write the constraints for the system.

2. Enter the equations into the <**Y=**> menu, select style, and determine an appropriate viewing window and scale. (Review Activity One for the steps.) Write your equations and window below.

3. Identify the feasible region and its corner points. To calculate the intersection of Y_1 and Y_2 press <**GRAPH**> and then <**2nd**> <**TRACE**> (CALC). Next press <**5**> (intersect) and then <**ENTER**> <**ENTER**> <**ENTER**>.



- The intersection of the two *Y* functions with the line $x = 175$ must be done by pressing **<2nd>** **<TRACE>** (CALC) **<1>** (value). Then type 175 and press **<ENTER>**.

- **<▼>** will give the second equation intersection.

- Thus, the corner points of the feasible region are _____,

  _____, and _____.

Minimize the cost:

$C(x, y) = .6x + .25y$

C(_____,_____) = .6( )+ .25( ) = $_____

C(_____,_____) = .6( )+ .25( ) = $_____

C(_____,_____) = .6( )+ .25( ) = $_____

Maximize the service:

$N(x, y) = x + y$

N(_____,_____) = _____ + _____ = _____

N(_____,_____) = _____ + _____ = _____

N(_____,_____) = _____ + _____ = _____

- Therefore, to minimize cost, students should buy _____ footballs and _____ ribbons.

- To maximize service, students should buy _____ footballs and _____ ribbons.

Activity Three: Three-Variable Systems

Teacher Notes

The Spirit Club has $200 to spend on souvenir items to be given away at "Homecoming." They would like to buy enough items so the first 500 fans who arrive receive a football, a spirit ribbon, or a button. The footballs, which are more popular, cost 60 cents each; the spirit ribbons, 25 cents each; and the buttons are 40 cents each. They have decided on an equal number of buttons and ribbons. How many of each should they buy? A check should be made to ensure that the correct number of items have been distributed within the budget.

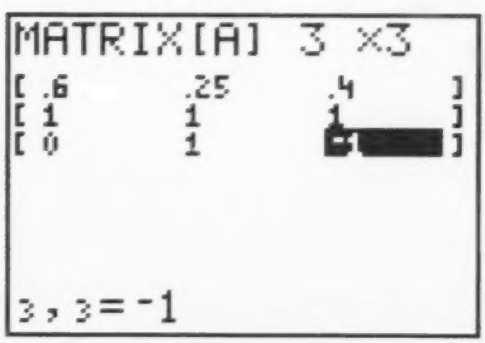
Matrix Solution

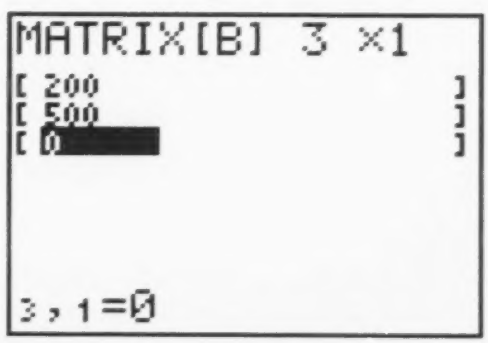
- Let x = number of footballs, y = number of ribbons, and z = number of buttons.

$$\begin{aligned} .6x + .25y + .4z &= 200 \\ x + \quad y + \quad z &= 500 \\ y - \quad z &= 0 \end{aligned}$$

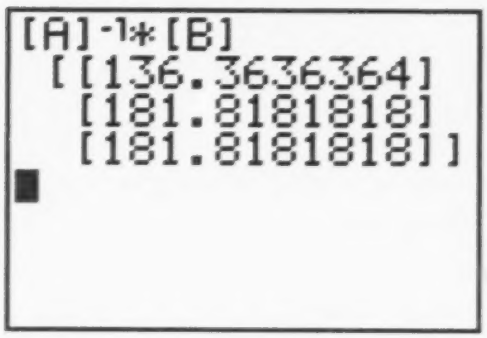
$$\begin{bmatrix} .6 & .25 & .4 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 200 \\ 500 \\ 0 \end{bmatrix}$$

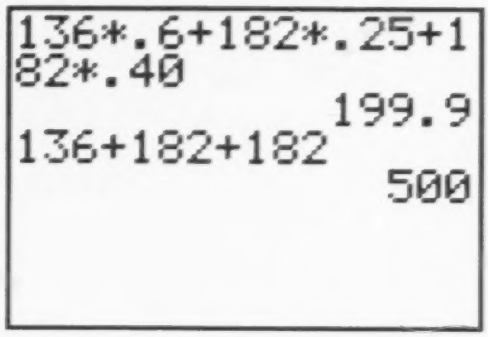
- To enter the coefficient as MATRIX [A], press <**MATRIX**> <▶> <▶> (EDIT) <**ENTER**> and change to a 3 x 3 matrix. Then enter the values.
- Press <**2nd**> <**QUIT**> to exit and save MATRIX [A].
- Then in a similar way enter the constant matrix as MATRIX [B]. Press <**2nd**> <**QUIT**> to exit and save MATRIX [B].





- Calculate the solution by solving for $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ using **<MATRIX>** **<ENTER>** (A).
- Then press $< x^{-1} >$.
- Next press **<X>** (Multiplication) and **<MATRIX>** **<2>** (B).
- Then press **<ENTER>** **< ENTER>**. Finally, perform the following calculations to determine the cost and total number of items the Spirit Club should order.





- Therefore, the students should purchase 136 footballs, 182 ribbons, and 182 buttons at a cost of $199.90.

Extension

- The Spirit Club has decided to sell footballs, ribbons, and buttons, instead of giving them away, in order to raise $500. They wish to sell twice as many footballs as ribbons. They also need the same number of ribbons as buttons.

1. How much should they charge for the footballs? ribbons? buttons?
2. Is this reasonable?
3. Would students be willing to buy items at this price?
4. What alternatives would you suggest to make the sale more successful?
5. How many of each would they have to sell to make $500 profit at the prices in #1? Assume the footballs still cost the Spirit Club 60 cents, the buttons 40 cents, and the ribbons 25 cents.

Activity Three Handout

The Spirit Club has $200 to spend on souvenir items to be given away at "Homecoming." They would like to buy enough items so the first 500 fans who arrive receive a football, a spirit ribbon, or a button. The footballs, which are more popular, cost 60 cents each; the spirit ribbons, 25 cents each; and the buttons are 40 cents each. They have decided on an equal number of buttons and ribbons. How many of each should they buy? A check should be made to ensure that the correct number of items have been distributed within the budget.

Matrix Solution

- Let x = number of footballs, y = number of ribbons, and z = number of buttons.

1. Write three equations in three unknowns to express algebraically the problem information.

2. Write the coefficient and constant matrices in below.

$$\begin{bmatrix} \quad \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

- To enter the coefficient as MATRIX [A], press <**MATRIX**> <▶> <▶> (EDIT) <**ENTER**> and change to a 3 x 3 matrix. Then enter the values. Press <**2nd**> <**QUIT**> to exit and save MATRIX [A].

- Then in a similar way enter the constant matrix as MATRIX [B]. Press <**2nd**> <**QUIT**> to exit and save MATRIX [B].

- Calculate the solution by solving for $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ using **<MATRIX>** **<ENTER>** (A).
- Then press $< x^{-1} >$.
- Next press **<X>** (Multiplication) and **<MATRIX>** **<2>** (B).
- Then press **<ENTER>** **< ENTER>**. Finally, perform the calculations to determine the cost and total number of items the Spirit Club should order.

- Therefore, the students should purchase _____ footballs, _____ ribbons, and _____ buttons at a cost of $_____.

Extension

- The Spirit Club has decided to sell footballs, ribbons, and buttons instead of giving them away in order to raise $500. They wish to sell twice as many footballs as ribbons. They also need the same number of ribbons as buttons.

1. How much should they charge for the footballs? ribbons? buttons?

2. Is this reasonable?

3. Would students be willing to buy items at this price?

4. What alternatives would you suggest to make the sale more successful?

5. How many of each would they have to sell to make $500 profit at the prices in #1? Assume the footballs still cost the Spirit Club 60 cents, the buttons 40 cents, and the ribbons 25 cents.

Packet One: Basic Calculus on the Graphing Calculator

Author

★ Eddie Keel, Haywood HS
Brownsville, TN

Approximate Time Needed for Packet

Approximately 20-30 minutes for each of the activities.

Content Area

Patterns, Functions, and Algebraic Thinking

Process Standards

Connections within Mathematics

Goals for Students

- To find the numerical derivative of a function at a point
- To graph a derivative
- To find the roots of a derivative
- To find solutions of equations
- To solve equations involving derivatives
- To find the definite integral of a given function

Description or Summary of Activity Packet

Students in calculus are the intended audience. Students will work individually and use the TI-82 or TI-83 to reinforce and enhance basic calculus concepts. This packet comprises six separate activities to enhance each of the concepts as it is studied during the calculus course. The activities teach familiarity with using the graphing calculator to perform the above calculations and to find the graphic solution to enhance the algebraic solution. The packet may also be used to review some of the key concepts of calculus.

Background Information

Prior to the included activities, students will have studied basic material for each of the concepts and will have mastered substitution, graphing, and solving equations. Also, students will have knowledge of basic keystrokes on the graphing calculator.

Materials and Equipment

- View screen with graphing calulator
- Overhead projector
- Each student will need:
  - access to a TI-82 or TI-83 graphing calculator
  - several sheets of graph paper and a straight edge
  - handouts

Activities One through Six

Teacher Notes

As each activity is used in the packet, the teacher will demonstrate the graphing calculator keystrokes and procedures as described in the example. Then the students will work individually with the graphing calculator to complete the appropriate handout.

Activity One:

Finding the Numerical Derivative of a Function at a Point—Activity One Handout

Activity Two:

Graphing the Derivative of a Function—Activity Two Handout

Activity Three:

Finding the Roots of a Derivative—Activity Three Handout

Activity Four:

Finding Solutions of Equations—Activity Four Handout

Activity Five:

Solving Equations Involving Derivatives—Activity Five Handout

Activity Six:

Finding the Definite Integral of a Given Function—Activity Six Handout

Activity One Handout

Finding the Numerical Derivative of a Function at a Point

I. TI Commands

$$\text{If } f(x) = x^3 - x^2, \text{ find } f'(3)$$

- On the TI-82 or TI-83, you will want to write this as nDeriv$\left(x^3 - x^2, x, 3\right)$. "$x^3 - x^2$" refers to the function whose derivative you want to find. x is the variable that you are using in the equation. 3 is the x-value of the point where you want to find the derivative.

Note: This is not correct calculus notation! This is simply the command you must type in the calculator at the appropriate screen (see below) to get the answer.

- Complete the following. Pay attention to parentheses in #3 and #4.

1. What command will you use to tell the TI-82 or TI-83 to calculate $f'(2)$ if $f(x) = 5x + 3$?

2. What command will you use to tell the TI-82 or TI-83 to calculate $f'(-1)$ if $f(x) = x^5$?

3. What command will you use to tell the TI-82 or TI-83 to calculate $f'(5)$ if $f(x) = \sqrt{3x - 5}$?

4. What command will you use to tell the TI-82 or TI-83 to calculate $f'(-2.25)$ if $f(x) = \dfrac{3x - 4}{x}$?

II. Keystrokes

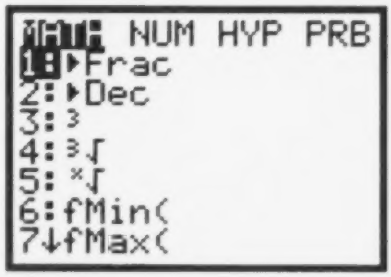
- Once you know what command to type on your graphing calculator, you must learn *how* to type the command.

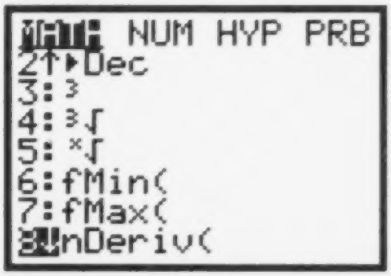
1. Push the <**MATH**> key.
2. Press the < $\blacktriangledown$ > key until you get to **8** for nDeriv(.
   Press <**ENTER**>.
   (Shortcut: Press <**8**> to get nDeriv(, and you will not have to scroll down with the down arrow key).
3. Type the rest of the nDeriv(command (that is in the parenthesis).
4. Press <**ENTER**>. The answer should appear on the screen.

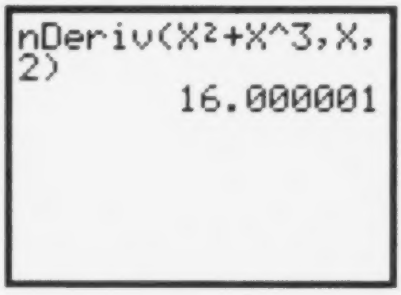
- *Try this procedure with the following example.*

III. Example

$$f'(2) \text{ if } f(x) = x^2 + x^3$$







IV. Problems

- In the following problems, find the numerical derivative at the point provided. In addition, write the command that you typed on your graphing calculator.

1. $f'(1)$ if $f(x) = x^4 + x^2$

(a) command _____ (b) solution _____

2. $f'(-1)$ if $f(x) = x - 4x^{12}$

(a) command _____ (b) solution _____

3. $f'(5)$ if $f(x) = \sqrt{x}$

(a) command _____ (b) solution _____

4. $f'(-2.25)$ if $f(x) = \dfrac{3x^2 - 5}{x - 3}$

(a) command _____ (b) solution _____

5. $f'(-132.856)$ if $f(x) = \dfrac{x}{x + 3}$ 206

(a) command _____ (b) solution _____

Activity Two Handout

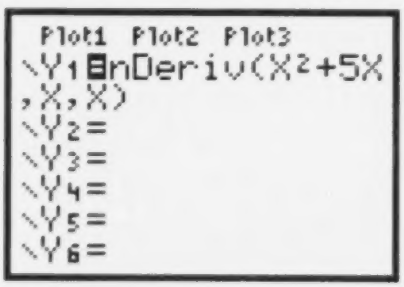
Graphing the Derivative of a Function

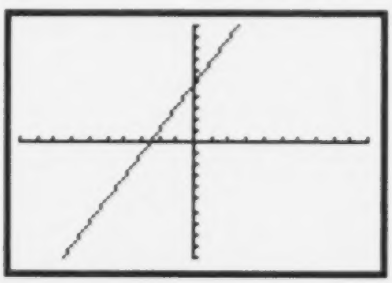
I. Procedure

1. Press the <**Y=**> key.
2. Press <**MATH**> <**8**> to get nDeriv(.
3. Type the function, x, and x. Use commas to separate these expressions and close the parentheses.
4. Graph the function using any method you prefer. <**ZOOM**> <**6**> (Standard) will work on many problems. However, the window may need adjusting on some problems.

II. Example

- Use the procedure above to Sketch the derivative of $f(x) = x^2 + 5x$.





III. Exercises

- Using your graphing calculator, sketch the derivatives of the functions below.

1. Sketch the derivative of $f(x) = 2x$.

2. Sketch the derivative of $f(x) = -2x + 4$.

3. Sketch the derivative of $f(x) = 9 - x^2$.

4. Sketch the derivative of $f(x) = x^3 - 4x$.

5. Sketch the derivative of $f(x) = x^4 - 5x^3$.

IV. For Further Thought

6. Sketch the derivative of $f(x) = \dfrac{2x-3}{x^2-16}$.

7. Describe the appearance of this graph. Why does it look this way?

8. Calculate the derivative algebraically. See if this answer matches your answers in #6 and #7.

Activity Three Handout

Finding the Roots of a Derivative

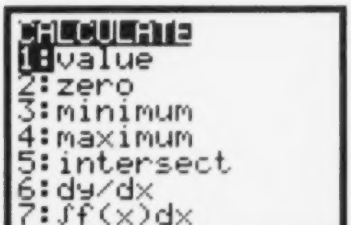
I. Procedure

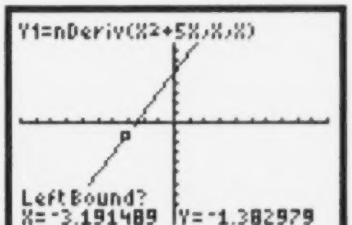
1. Graph the derivative (see previous activity).
2. Press <**2nd**> <**TRACE**> (CALC).
3. Press <**2**> (for zero on a TI-83 or root on a TI-82) . Then, press <**ENTER**>.
4. You will see a "Left Bound?" or "Lower Bound?" question and a blinking cursor. Using the <◁ > or < ▷> key, move the cursor so that it is to the left of the root. (Keep in mind that sometimes this is above the x-axis and sometimes this is below the x-axis. This is irrelevant. The cursor must be to the left of the root.)
5. Press <**ENTER**> when you have finished moving the cursor.
6. You will now see a "Right Bound?" or an "Upper Bound?" question. Using the <◁ > or < ▷> key, move the cursor so that it is to the right of the root. (Keep in mind that sometimes this is above the x-axis and sometimes this is below the x-axis. This is irrelevant. The cursor must be to the right of the root.)
7. Press <**ENTER**> when you have finished moving the cursor.
8. You will see a "Guess?" question. Move the cursor so it is close to the root. Then, press <**ENTER**>.
9. The solution should now appear on the screen.

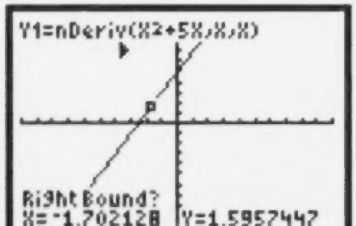
II. Examples

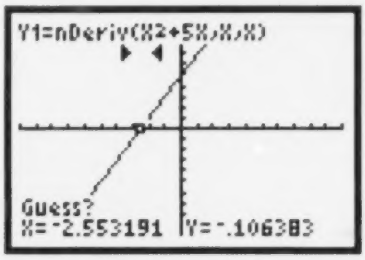
- Use the procedure above to solve the following question:

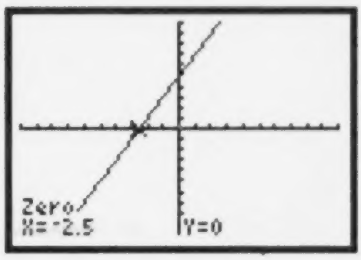
$$\text{If } f(x) = x^2 + 5x, \text{ find all values of } x \text{ such that } f'(x) = 0.$$



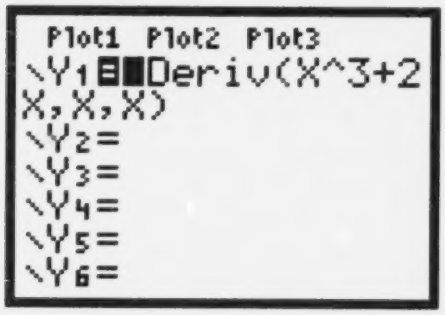


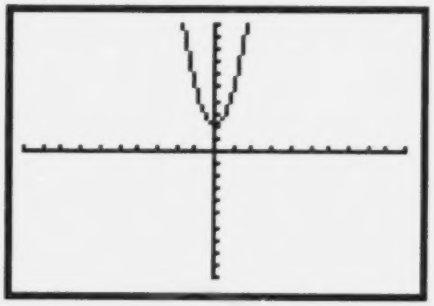




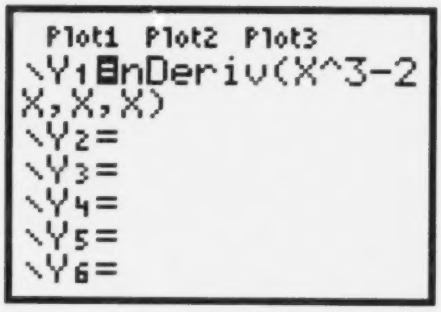


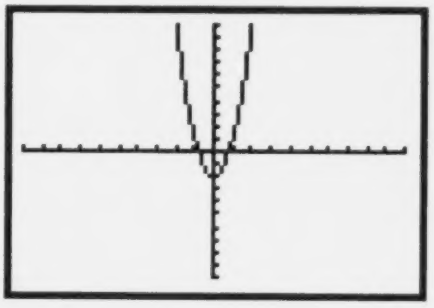
- Some derivatives do not have roots. Look at the graph of the derivative of $f(x) = x^3 + 2x$. Why doesn't it have any roots?





- Some derivatives have more than one root. Look at the derivative of $f(x) = x^3 - 2x$. The graph of the derivative crosses the x-axis at two different points. You will have to perform the procedure twice to find both roots. Find the left root first and then find the right root. The two solutions should be x is approximately ± 0.816.





III. Exercises

1. If $f(x) = 5x^2 + 8$, then find all values x such that $f'(x) = 0$.

2. If $f(x) = x^3 - 10x^2$, then find all values x such that $f'(x) = 0$.

3. If $f(x) = x^3 + 8x$, then find all values x such that $f''(x) = 0$.

4. If $f(x) = x^3 - 8x$, then find all values x such that $f''(x) = 0$.

5. If $f(x) = 9x^3 - 8x^2 - 7x$, then find all values x such that $f''(x) = 0$.

Activity Four Handout

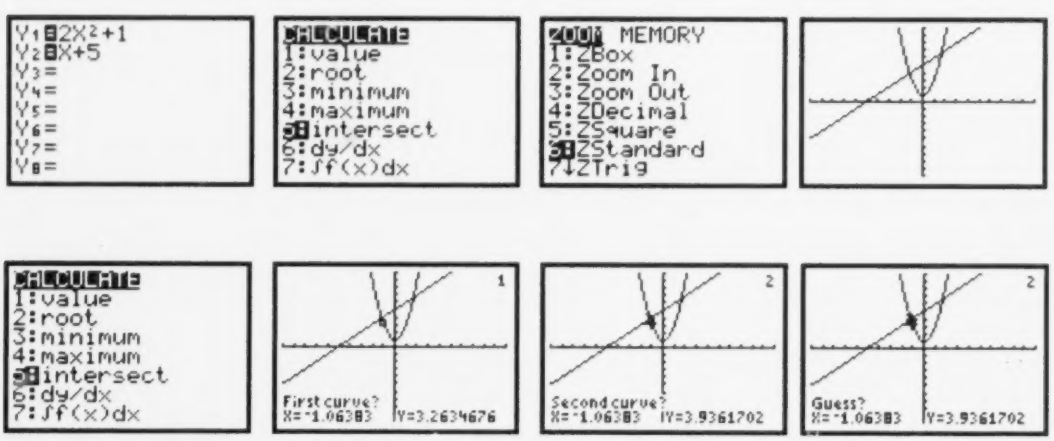
Finding Solutions of Equations

I. Procedure

1. Press <**Y=**>. Type the equation on the left of the equal sign in Y_1.
2. Type the equation on the right of the equal sign in Y_2.
3. Press <**ZOOM**> <**6**> for the standard viewing screen. If you cannot see good pictures of the two graphs and their points of intersection, you will need to adjust the Window settings. To do this press the <**Window**> key and change the Xmin, Xmax, Ymin, and Ymax settings.
3. Press <**2nd**> <**CALC**>.
4. Press <**5**> for Intersect.
5. Determine the number of points of intersection. If there are more than one, you will have to repeat this procedure more than once.
6. A "First Curve?" message will appear on the screen. Use the <◁ > or < ▷> key to move to the point of intersection you want to find. Press <**ENTER**>.
7. A "Second Curve?" message will appear on the screen. Use the <◁ > or < ▷> key to move to the point of intersection you want to find. Press <**ENTER**>.
8. A "Guess?" message will appear on the screen. Use the <◁ > or < ▷> key to move to the point of intersection you want to find. Press <**ENTER**>.
9. The answer will now appear.

II. Example

- Solve the following equation for x: $2x^2 + 1 = x + 5$.



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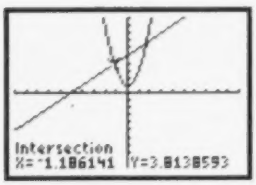

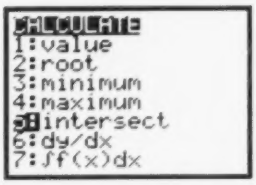

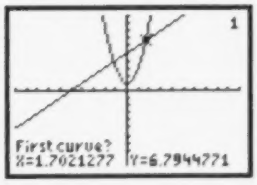

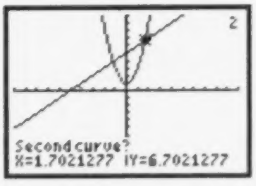

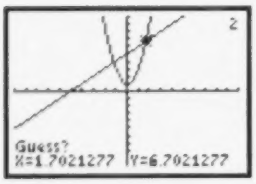

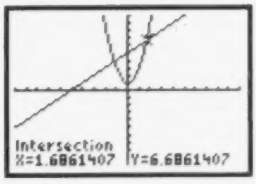

III. Exercises

- Solve for x using a graphing calculator. Make sure you provide the following documentation:
  a. sketch the graphs of both equations on one grid using your graph paper
  b. write down the Window that you used
  c. label the point of intersection
  d. provide the ordered pair of the point of intersection
  e. write the answer to the problem

1. $2x + 3 = x^2$

2. $9 - x^2 = x^2$

3. $x^3 - 5x = 5$

4. $x^3 - 5x = 2x + 1$

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5. $x^4 - 3x^2 - x = 2x - 4$

Activity Five Handout

Solving Equations Involving Derivatives

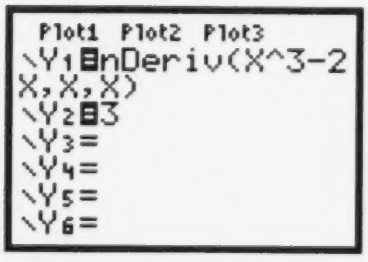
I. Procedure

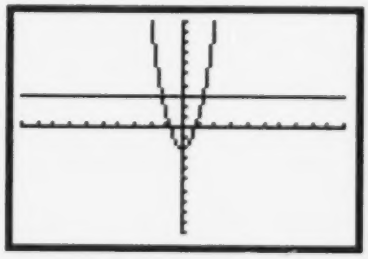
- Most types of algebraic problems in the form $f(x) = g(x)$ that you want to solve for x given $f'(x) = h(x)$ can be solved using the following procedure on the TI-82 or TI-83.

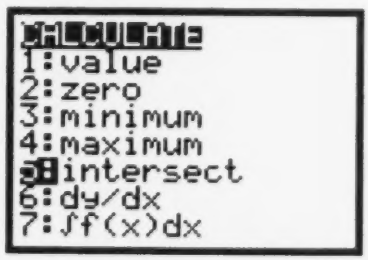
1. Type nDeriv($g(x), x, x$) in Y_1 .
2. Type $h(x)$ in Y_2 .
3. Press **<ZOOM> <6>** for the standard viewing screen. If you cannot see a good picture of the two graphs and their points of intersection, you will need to adjust the Window settings. To do this, press **<WINDOW>** and change the Xmin, Xmax, Ymin, and Ymax settings.
3. Press **<2nd> <TRACE>** (CALC).
4. Press **<5>** for Intersect.
5. Determine the number of points of intersection. If there are more than one, you will have to repeat this procedure more than once.
6. A "First Curve?" message will appear on the screen. Use the <◁ > or < ▷> to move to the point of intersection you want to find. Press **<ENTER>**.
7. A "Second Curve?" message will appear on the screen. Use the <◁ > or < ▷> key to move to the point of intersection you want to find. Press **<ENTER>**.
8. A "Guess?" message will appear on the screen. Use the <◁ > or < ▷> key to move to the point of intersection you want to find. Press **<ENTER>**.
9. The answer will now appear.

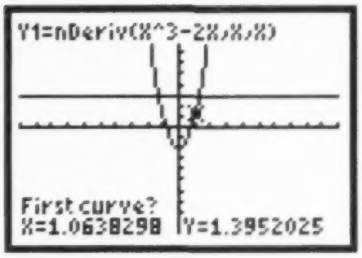
II. Example

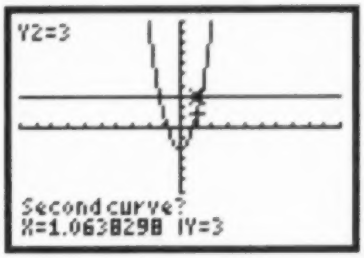
- Let $f(x) = x^3 - 2x$. Find all values of x such that $f'(x) = 3$.

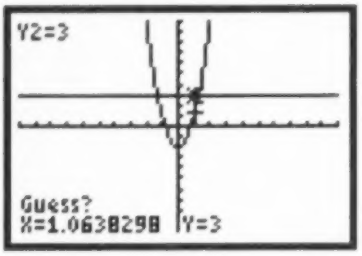


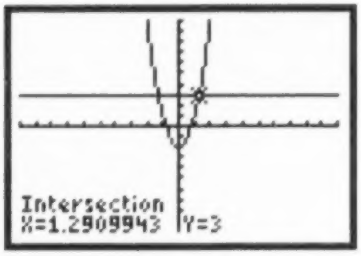




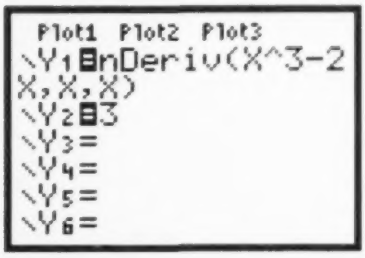


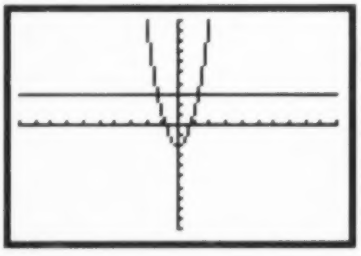


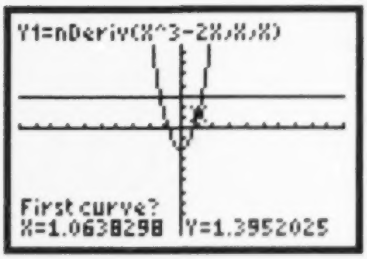


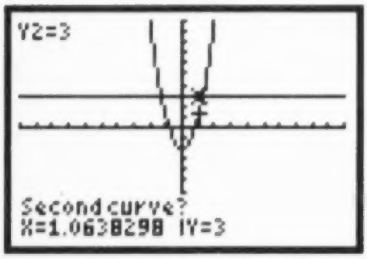


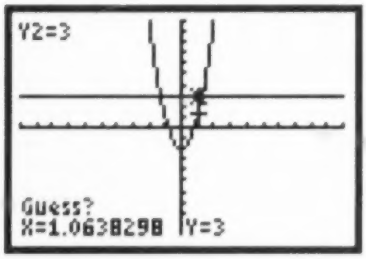
- When you repeat the procedure to get the 2nd intersection point, this is what the procedure looks like on a TI-83:

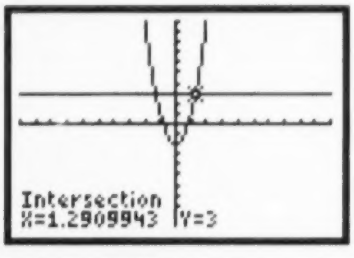


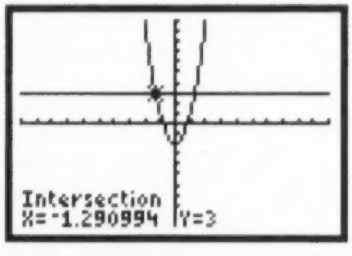












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III. Exercises

- Provide appropriate documentation as in Activity Four.

1. Let $f(x) = x^2$. Find all values of x such that $f'(x) = 5$.

2. Let $f(x) = x^3$. Find all values of x such that $f'(x) = 2$.

3. Let $f(x) = x^3 - 5x^2 - 10x$. Find all values of x such that $f'(x) = 1$.

4. Let $f(x) = 9x - x^3$. Find all values of x such that $f'(x) = -3$.

5. Let $f(x) = x^3 - 2x^2$. Find all values of x such that $f'(x) = x$.

Activity Six Handout

Finding the Definite Integral of a Function

I. TI Commands

$$\int_3^5 x^4 dx$$

- On the TI-82 or TI-83, you will want to write this as fnInt(x^4, x, 3, 5) . x^4 refers to the function of which you want to find the integral. x is the variable used in the equation. 3 is the lower limit on the integrand and 5 is the upper limit on the integrand.

Note: This is not correct calculus notation! This is simply the command you must type in the calculator to get the answer.

1. What command will you use to tell the TI-82 or TI-83 to calculate $\int_0^1 x^2 dx$?

2. What command will you use to tell the TI-82 or TI-83 to calculate $\int_0^3 (x^3 - 5x^2 + 6x) dx$?

3. What command will you use to tell the TI-82 or TI-83 to calculate $\int_{-2}^{3} \frac{x^2 - 9x}{2x} dx$?

4. What command will you use to tell the TI-82 or TI-83 to calculate $\int_0^2 \sqrt{x} dx$?

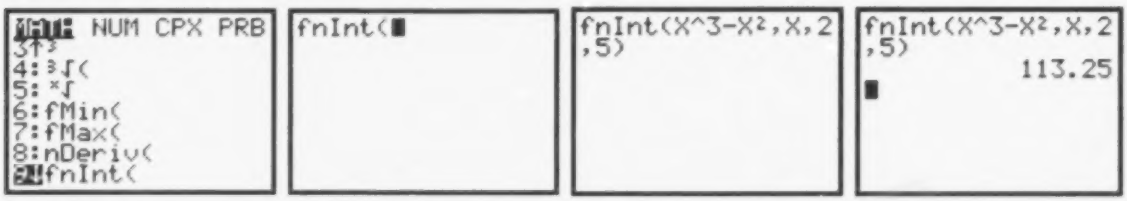
II. Keystrokes

- Once you know what command to type on your graphing calculator, you must now learn *how* to type the command.

1. Push the <**MATH**> key.
2. Press the <▼>key until you get to 9 for fnInt(. Press <**ENTER**>. (Shortcut: Press 9 to get fnInt(and you will not have to scroll down with the down arrow key.)
3. Type the rest of the fnInt(command (as described in Section I above).
4. Press <**ENTER**>. The answer should appear on the screen.

III. Example

- Try this procedure with the following example: $\int_2^5 (x^3 - x^2)dx$



IV. Exercises

- In the following problems, find the definite integral. In addition, write the command that you typed on your graphing calculator. Keep in mind that these answers are sometimes approximations.

1. $\int_0^2 x dx$

   a. command _____ b. solution_____

2. $\int_0^2 x^2 dx$

   a. command _____ b. solution_____

3. $\int_0^2 x^3 dx$

   a. command _____ b. solution_____

4. $\int_0^2 \left(\frac{1}{x+1}\right) dx$

   a. command _____ b. solution_____

5. $\int_{-10}^{10} \left(\frac{x^2 - 5x}{x^2 + 8}\right) dx$

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   a. command _____ b. solution_____

Packet Two: Calculator-Based Lab Activities

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Approximate Time Needed For Lesson

One to two traditional class periods or one 90-minute period for each of the activities

Content Area

Probability and Statistics and Patterns, Functions, and Algebraic Thinking

Process Standards

Problem Solving and Connections

Goals for Students

- To collect and analyze real-world data
- To model real-world phenomena using functions and graphs

Description or Summary of Activity Packet

Activities One and Three (Bouncing Balls and Swimming Pool) are appropriate for Algebra I, Math Tech II, and Algebra II students. All three activities are appropriate for Pre-Calculus Students. Using a Calculator Based Lab (CBL) and probes, students will collect and analyze real-world data. This packet comprises three separate activities. One collects data from a bouncing ball, one collects data from a swinging pendulum, and one collects data from a "swimming pool." Beginning algebra students will use this data to work with parabolic functions, and more advanced students will work with exponential functions. Work can be accomplished successfully in groups, but entire class demonstrations could also be useful. The activities can be completed all together and one at a time, depending upon the mathematical background of the students.

Background Information

Prior to the included activities, the students will have had experience with basic key strokes and with finding linear and quadratic regression equations on the graphing calculator. The students will have also studied linear and non-linear functions.

Materials

- Graph-Link
- Graphing calculator with view screen and projector, if available
- Each student will need:
  - √ access to a CBL
  - √ access to a motion detector (Activities One and Two) and a pressure probe (Activity Three)
  - √ access to a TI-83 graphing calculator with link cable
  - √ handouts
- Additional materials listed for each activity

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Activity One: Graphing the Time vs. Height Curve for a Bouncing Ball

Teacher Notes

Additional Equipment:

√ Basketball

This activity requires a graphing calculator program to input the data collected by a CBL. Such a program may be any generic one that collects data from the motion detector for about 10 seconds or the bounce program that comes with the CBL. Appropriate ones may be downloaded from the TI web site (http://www.ti.com/calc/docs/cblwb3.htm).

- Two good programs are available from the sources listed below.
  - *Real-World Math with the CBL System* by Brueningsen, Bower, Antinone, and Brueningsen, published by Texas Instruments, Activity 9: That's the Way the Ball Bounces
  - *CBL Explorations in Calculus for the TI-82* by Nichols, published by Meridian Creative Group, Activity 8: Analyzing a Bouncing Ball
- As technology advances, more calculators will be useful. Certainly, the TI-82 and TI-83 are already available. A possible advantage to using a person to hold the motion detector: if the ball does not bounce straight up and down, the person can follow the path of the ball, making sure to keep the height of the motion detector constant.
- Extension: After Question 4 in Handout 1, the following extension may be assigned. Students repeat the process of finding parabolic functions for each of the parabolic sections. Students can then examine the relationships between these functions. If this process is to be used, care must be taken to preserve all original data.
- Handout 1 includes alternate methods for finding the function. It is possible to challenge students by letting them initially work, using $y = ax^2 + bx + c$. Helpful ideas are as follows:

  c= y-intercept, the sum of the roots $= -\frac{b}{a}$, the product of the roots $= \frac{c}{a}$.
- Different balls and different initial heights may be used to see how this alters the functions. Similar experiments can be performed with other objects such as a yo-yo.

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Suggestions for Assessment:

- Teachers will certainly assess student learning by monitoring student progress while the class is working in groups to perform the experiment and answer the questions. It is recommended that students then write and/or orally present this lab report for further assessment by the teacher.

Suggestions for Staff Development:

- Staff development activities should include hands-on activities so that teachers will truly experience these activities. Teachers should be encouraged to interact with one another, assist one another, and share ideas.

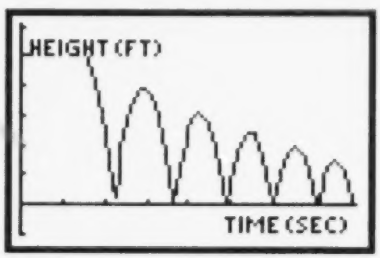
Activity One, Handout 1

- The motion detector must be positioned above the ball. Note: Try various balls to see what will yield good data. Balls must be large enough to be clearly picked up by the motion detector. Balls with fuzzy surfaces, such as tennis balls, often do not yield good data.

- You may attach the motion detector to a stationary support such as a ring stand, or you may let a person hold the detector at a constant height with the person's arm extended. The ball should be dropped from a position that is at least 18 inches below the motion detector. Be careful that the person or equipment does not interfere with the path between the motion detector and the ball. Make sure the motion detector is in place facing down and is connected to the CBL that is connected to the graphing calculator.

  1. Begin the program.
  2. Drop the ball as soon as clicking is heard.
  3. Check to see that if good results were obtained. (If not, run the experiment again.)

The data should yield a graph similar to this sample.



- First, study one parabolic section from your graph. Just select one and read from the graph, but if a program is available to select one section, use it. The program will probably put the selected data in two particular lists (doing this by hand is tedious), so take note of which two lists are used.

1. The ball bounced mostly straight up and down. Why does the graph look like it was bouncing from left to right? What values compose the x-coordinates and the y-coordinates?

2. Using trace or the lists, estimate the vertex of the parabola. (Round decimals to hundredths.)

3. The vertex form of a parabolic expression is given by

$$y = a(x - h)^2 + k$$

where *a* is a constant and the coordinates of the vertex are (h, k). h and k were found in Question 2. You will need to make an estimate of the value for *a*. (Something close to -1 is probably a good first choice.) Now use the **<Y=>** key on your calculator to type in the expression $a(x - h)2 + k$ using the determined values for a, h, and k for Y_1. Press **<GRAPH>**. Make adjustments for the value of *a* until you obtain a good fit. Write your equation below.

4. What affect does the sign and magnitude of *a* have on the graph?

Extension: You may repeat this process with other parabolic sections and examine the relationships between their algebraic functions.

5. Algebraically transform your function in question 3 to the form $y = ax^2 + bx + c$ and state it below.

6. Now use quadratic regression in your calculator on the same parabolic section of your data. Write the regression equation below.

7. Are the answers in questions 5 and 6 consistent? Explain.

Activity One, Handout 2

1. Perform the experiment as listed in Handout 1 again, or use the data collected in Handout 1. Use trace or the lists to find the height in meters at the beginning of the experiment and at the top of the vertex of each parabola. Let the *x*-coordinates denote the number of the bounce with the initial position having an *x*-coordinate of 0, and the vertex of the first parabola having an x-coordinate of 1, etc. The *y*-coordinate will be the height. Enter this data in two available lists in your calculator.

2. Use your calculator to construct a scatter plot representing this data. Sketch the plot below.

3. Use your calculator to compute a linear regression equation for this data.

4. Type the linear regression equation in <**Y=** > and graph it. Sketch the graph below.

5. Does this seem to be a reasonable representation of the data? Explain.

6. Now use your calculator to compute an exponential regression on this data.

7. Type this regression equation in <**Y=**> (eliminate or turn off the linear regression) and graph it.

8. Is this a reasonable representation of this data? Explain.

9. Use your data or graph to find the initial height of the ball and the height reached after each bounce to as many bounces as are contained in your data. Then use the regression equation in Question 7 to compute the height after the next few bounces until you have values for the original height and the height reached after the first 10 bounces. (You may wish to round values to hundredths.)

10. How far has the ball traveled from the time it was dropped until it reaches its maximum height after the tenth bounce?

Activity Two: Graphing the Movement of a Pendulum

Teacher Notes

Additional Equipment

- √ Pendulum bob and string
- √ Stopwatch
- √ Meter stick
- √ Support such as a ring stand to be used to attach pendulum

In this activity students will use a CBL and motion detector to obtain data demonstrating the movement of a pendulum. Students will then develop a trigonometric function that approximates this sinusoidal curve. Students will develop this work further to find and examine graphs for velocity and acceleration using derivatives.

- This experiment requires a program for the graphing calculator. Such a program may be any generic one that collects data from the motion detector or the pendulum program that comes with the CBL. Appropriate ones may be downloaded from the TI website address: (http://www.ti.com/calc/docs/cblwb3.htm)

- Two good programs are available from the sources listed below.

  - *Real-World Math with the CBL System* by Brueningsen, Bower, Antinone, and Brueningsen, published by Texas Instruments, Activity 16: Tic, Toc

  - *CBL Explorations in Calculus for the TI-82* by Nichols, published by Meridian Creative Group, Activity 10: Analyzing a Pendulum

- Physical measurements are used in appropriate places. Values can also be obtained from the graph. The choice is left to the teacher's discretion.

- Question 8 allows great opportunity to discuss concepts from calculus such as the derivative of a constant is zero and the chain rule.

- Question 10 should allow good discussion of the relationships between the cosine and sine functions in differentiation. Also of importance are the concepts of velocity and acceleration as the pendulum changes position.

Suggestions for Assessment:

- Teachers will certainly assess student learning by monitoring student progress while the class is working in groups to perform the experiment and answer the questions. It is recommended that students then write and/or orally present this lab report for greater assessment by the teacher.

Suggestions for Staff Development:

- Staff development activities should include hands-on activities so that teachers will truly experience these activities. Teachers should be encouraged to interact with one another, assist one another, and share ideas.

Activity Two Handout

- Hang the pendulum and position the motion detector so it is at approximately the same height as the pendulum bob and so the motion of the pendulum will swing straight forward and backward from the motion detector. Raising the pendulum and motion detector above a flat surface (table or floor) may help remove interference from that flat surface.

- Before you begin your program, make the following measurements.

  1. Distance from the motion detector to the pendulum in its stationary position: _____cm.
  2. Distance the pendulum will be pulled back from its stationary position: _____cm.
  3. Time for 10 complete cycles of the pendulum: _____sec.
     (One cycle consists of one complete swing forward and back.)

- Now run the program. One person will need to position the pendulum so it is pulled back the planned distance. This person must be ready to move out of the way of the motion detector as soon as the pendulum is released. It is best to have a different person operate the calculator. Make sure the motion detector, CBL, and calculator are connected and everything is turned on.

  1. Decide on a signal so both people coordinate their activities at the beginning of the experiment.
  2. Simultaneously release the pendulum from its pulled-back position and begin collecting data on the calculator.
  3. Check to see if good results were obtained. (If not, run the experiment again.)

- Your graph should be sinusoidal. Now fit this curve to a function of the form

$$y = A\cos B(X - C) + D$$

using the following questions.

1. A represents the amplitude, which is the distance from a high point on the curve to the center horizontal axis of the curve. What is the amplitude of your curve?

   A = _____cm.

2. B represents the number of cycles your sinusoidal curve makes during the natural period of the cosine function (2π).

   a. The period of your curve = $\frac{\text{time for 10 cycles}}{10}$ sec.

   b. Find B by dividing 2π by your answer in part a above. B = _____

3. C represents the horizontal shift of your curve. Since the cosine function reaches a maximum when it intersects the y-axis, use trace with any maximum value of your curve to determine C.

   C = _____

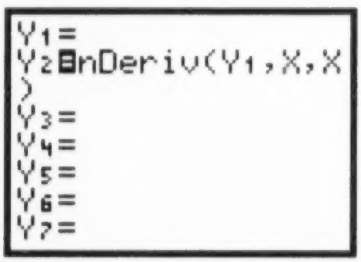
4. D represents the vertical shift of your curve from the x-axis. This is the distance between the stationary pendulum and the motion detector. D = _____cm.

5. Now using <**Y=**> type your numbers in the function y = AcosB(X-C) + D for Y_1.

6. Graph Y_1 simultaneously with your data. Is your equation an acceptable fit to your data? If not, analyze your work and make adjustments. Write your equation that produces a good fit.

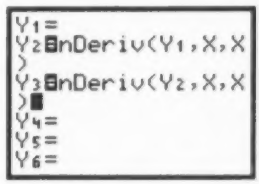
- Now you can extend your investigation of this data into the area of calculus.

7. Use nDeriv(to graph the first derivative of your function in Y_2.
   Hint: nDeriv(is found as line 8 in the <**MATH**> menu.
   Y_2 should be typed like the screen below shows. (Of course you will have your function in Y_1.) Remember the Y_1 names the function to be differentiated, the first x names the variable, and the second x directs the calculator to differentiate for all values of x.



8. Why does the graph of the derivative need such a different window? Why is Y_2 so much lower than Y_1? Why is the amplitude so different?

9. Use nDeriv(to graph the second derivative of your function in Y_1.
   Hint: Y_3 should be typed like the screen to the right.
   (Again you will have your function in Y_1.)



10. Discuss the relationships between the three functions.
    What are other names for the derivatives?
    What do these functions mean concerning the pendulum?

Activity Three: Pressure vs. Depth Curve for a Swimming Pool

Teacher Notes

Additional Equipment:

- Vernier pressure sensor
- Piece of plastic tubing at least 20 feet long (aquarium air tubing works well)
- Means of attaching weight (paper clip works well)
- Weight heavy enough to pull the tubing to the bottom of the water (a large fishing sinker works well)
- Permanent marking pen
- Ruler
- Long pipe to hold water
- Water

Students will use a CBL and pressure probe to measure water pressure as depth changes. This experiment will simulate the change in water pressure in a swimming pool as the depth changes. This experiment will use a tall pipe to simulate the pool. The resulting function will be linear.

- A convenient way to make the "swimming pool" is to use white plastic pipe that can be purchased at a hardware store. This pipe can be in sections that screw together. This will make storage and transport easier. You can also have some fun with your students by making it their duty to figure out how to empty the water at the end of the experiment.

- In order to perform this experiment the graphing calculator must have a program. If you are very proficient and wish to write your own program, that is fine. Most teachers will prefer to link calculators or use Graph-Link to download someone else's program. Your program must not only measure pressure, but it must also use the trigger on the CBL. One good program that is available is listed below.

  - **Real-World Math with the CBL System** by Brueningsen, Bower, Antinone, and Brueningsen, published by Texas Instruments, *Activity 3: Pool Plunge*

- If the pipe is too elaborate, a garden hose will also work. Of course you can always take a field trip to a swimming pool. Hawaii might be fun!

Suggestions for Assessment:

- Teachers will certainly assess student learning by monitoring student progress while the class responds during the experiment and while students answer the questions. Teachers will have further opportunity to assess student learning if the assignment concludes with a written and/or oral report. This could even include ideas such as a discussion of depths with pressures that are safe for swimming and the depth at which the Titanic rests.

Suggestions for Staff Development:

- Staff development should include hands-on activities. Teachers should be encouraged to interact with one another, assist one another, and share ideas.

Activity Three Handout

- When a person dives into a swimming pool, there is a noticeable change in the pressure on the eardrums as the person descends to the bottom of the pool. The purpose of this experiment is to develop the mathematical relationship between the depth of the water and the pressure that is felt. Instead of using a pool, students can produce the same effect by pouring water into a pipe.

- This experiment uses plastic pipe that is at least 2 inches in diameter. Make sure the pipe is sealed on the bottom and is as tall as is convenient in your classroom. Pour water into this pipe to simulate the water in a pool. Now that your "swimming pool" is ready, begin to set up your other equipment. Make a small hole in one end of the plastic tubing and attach the weight. Beginning at that end of the tubing, measure and mark off intervals of one foot until you have exceeded the depth of the water. Attach the other end of the plastic tubing to the pressure sensor.

  - √ Measure the initial pressure.
  - √ Your program must be one that is activated by the trigger.
  - √ Start your program.
  - √ Lower the tubing into the water one foot at a time, taking a reading for each foot of depth.
  - √ Examine your graph to make sure it is acceptable.
  - √ Answer the questions below.

1. Sketch the graph of your data below.

2. Does your data look linear? Use **<TRACE>** to find the coordinates of two of your data points.

3. Find the slope of the line passing through the two points in Question 2.

4. Now, in slope-intercept form, write the equation of the line passing through those points.

5. Use your calculator to graph your line on the graph of your data. How good a fit is your line?

6. Another way to obtain a line representing your data is to use linear regression in your calculator. Do this and write the resulting equation below.

7. Do the equations in Question 4 and Question 6 agree? Explain.

8. Now graph the equation in Question 6 on the graph with your data. How well does this graph fit your data?